Structure Analysis

3rd Civil

Slope deflection [Beam + Frame]
Direct stiffness [Truss + Beam + Frame]

Prepared By:

Eng. Islam Mohamed

Teaching assistant in May Engineering for
Reinforced Concrete & Structure Analysis

011 56 56 7899

Contact Us:

Facebook:

www.facebook.com/groups/PerfectTrainingAcademy

www.facebook.com/groups/eng.islam.mohamed

E-mail:

perfect.training14@yahoo.com

Revision

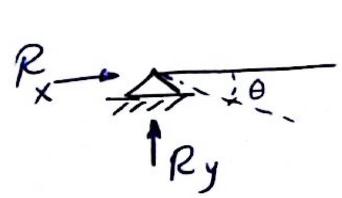
المنهج

- Slope Deflection
  - Beams
  - Frames
- Direct stiffness
  - Beams
  - Trusses
  - Frames
    - Without Axial Deformation
    - With Axial Deformation
- General stiffness
  - Beams

⇐ مواضيع مهمة لحل أي مسألة في أي طريقة :

① D.o.f

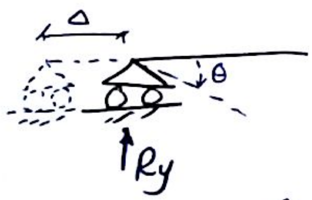
عدد الحركات المسموح بها عند النقطة



* $D.o.f = 1 \Rightarrow \theta$

Because

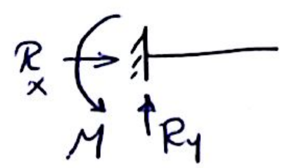
* $R = 2$ → R_y, R_x



$D.o.f = 2$ → θ, Δ

Because

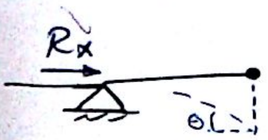
$R = 1 \rightarrow R_y$



$D.o.f = 0$

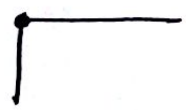
Because

$R = 3$ → R_x, R_y, M



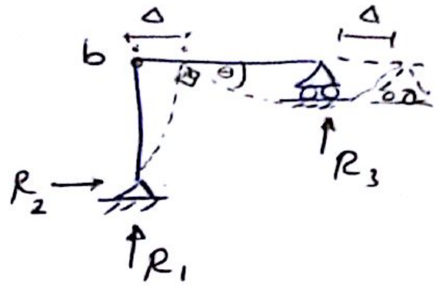
$D.o.f = 2$ → θ, Δy

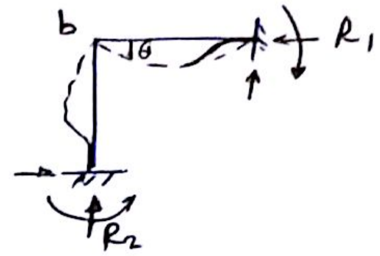
$R = R_x$ عند النقطة



لابد من معرفة Supports المتصلة بها لتحديد حركات العضو

Ex:





@b

$D.o.f = 2 \rightarrow \Delta x, \theta$

Because

$R = R_1 \rightarrow$ ينعدم بها الحركة الرأسية

← أما في اتجاه x فال (roller) لا

تمنع الحركة .

@b

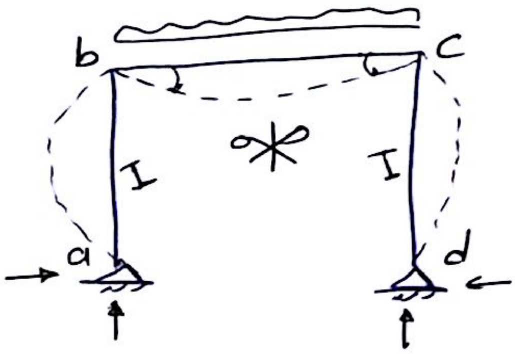
$D.o.f = 1 \rightarrow \theta$

Because

$R = 2 \rightarrow R_1, R_2$

R_1: تمنع الحركة الأفقية

R_2: تمنع الحركة الرأسية



$\Rightarrow D.o.f = 6 \rightarrow \theta_a, \theta_d, \Delta x_c, \Delta x_b, \theta_b, \theta_c$

$\therefore D.o.f = 1 \rightarrow \theta_b$

$\Downarrow$

ديه اللي هنحل بيها

لأن

$\theta_a, \theta_d \rightarrow M = 0$

$\Delta x \rightarrow$ مش ممكن

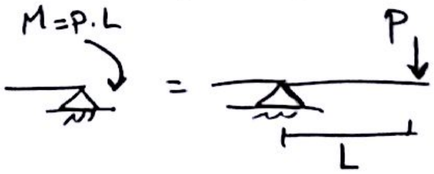
$\theta_b = -\theta_c \Rightarrow \theta_b$ نجيب

* يتم اهمال أي D.o.f

θ معلوم عندها العزم

سواء بصفر (hinge) أو

بقيمة



• في حالة التماثل لا يمكن حدوث

Δ وذلك لأن الأحمال ومقاومة

المنشأ متساوية فلا تميل ناحية

عن الأخرى .

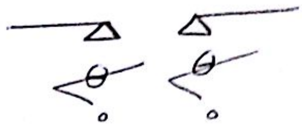
* في حالة التماثل تكون $\theta_b = -\theta_c$

هام :

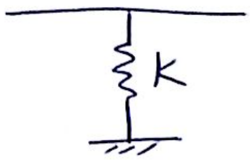
⇒

$D.o.f = 2 \begin{cases} \Delta y \\ \theta \end{cases}$

$\theta \rightarrow$ يمكن الفصل واعتبار modified وإهمال θ



⇒



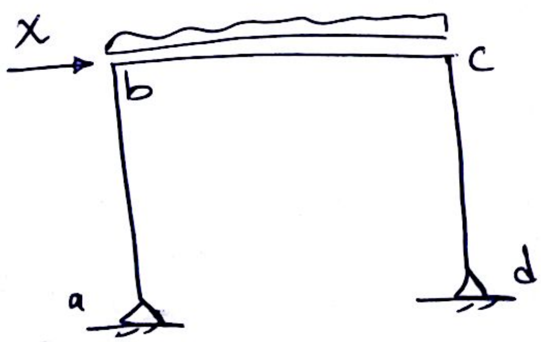
$D.o.f = 2 \begin{cases} \Delta y \\ \theta \end{cases}$

علشان ديه سوستة يعني هتنضغط

$D.o.f = 2 \begin{cases} \Delta x \\ \theta \end{cases}$

⇐ في حالة (Antisymmetric) التماثل العكسي :

المنشأ يكون متماثل في الأحمال ولكن جنب عليه أحمال مركزة مش موجودة في الآخر أو I للجنب غير التاني



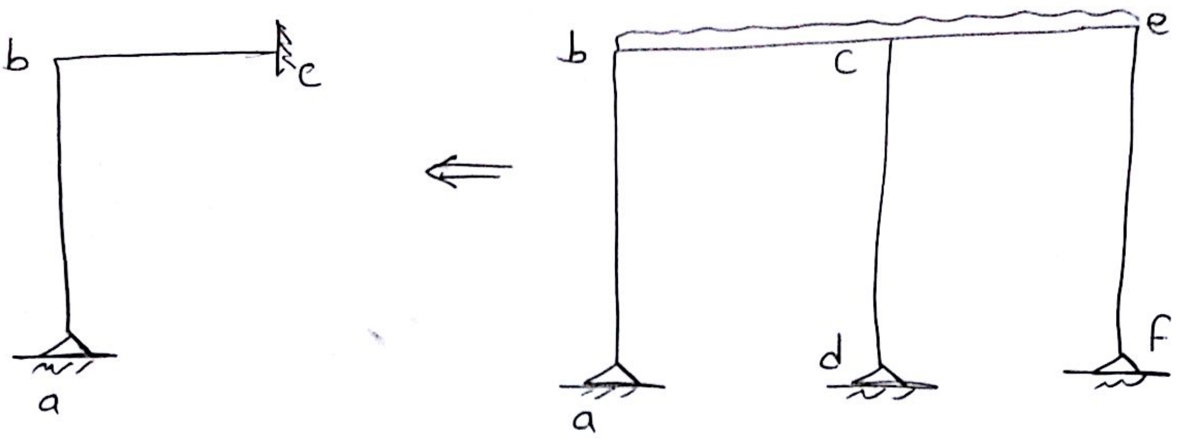
$D.o.f = 2 \begin{cases} \Delta \\ \theta_b \end{cases}$

يحدث Δ بسبب عدم التماثل

وتكون $\theta_c = \theta_b$

⇐ في حالة التماثل حول support يكون θ

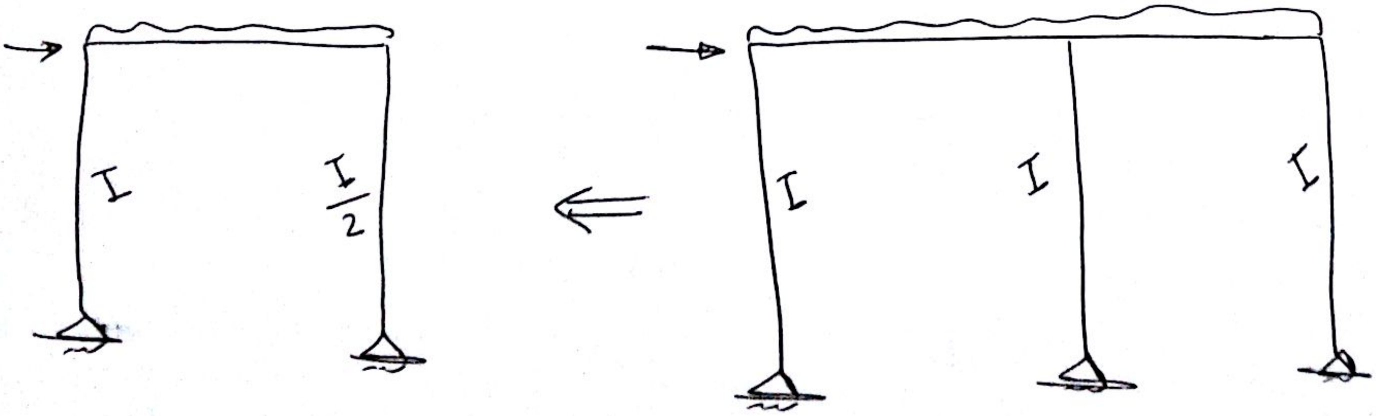
عندها بصفر أي يمكن الفصل واعتباره

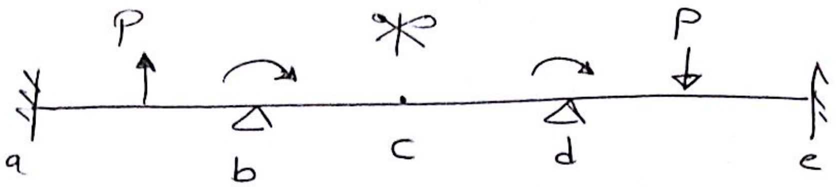


* لأن الأحمال متساوية فليس هناك سبب لدورانه جنب عن الآخر.

⇐ أما في حالة Anti-Symmetric (التماثل العكسي)

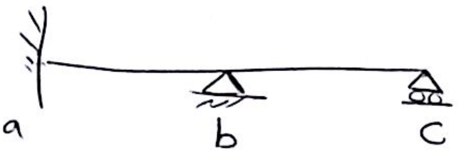
نأخذ نصف I للعنصر اللي حوليه التماثل أو فصل الكمرة بـ supports بتاعتها لو التماثل حول support معين أو وضع support والمفصل إذا كان التماثل حول نقطة وليس support.





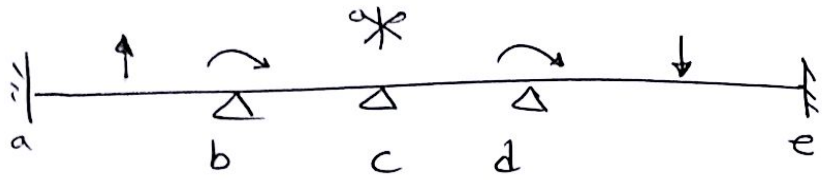
تماثل عكسي حول (C)

نعتبرها support ثم نفصل



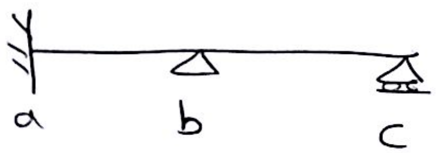
نحل الكمرة الجديدة

$D.o.f = \theta_b$



التماثل العكسي حول (C) ← support

يتم الفصل عنده

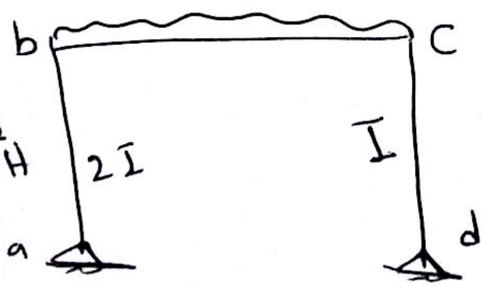


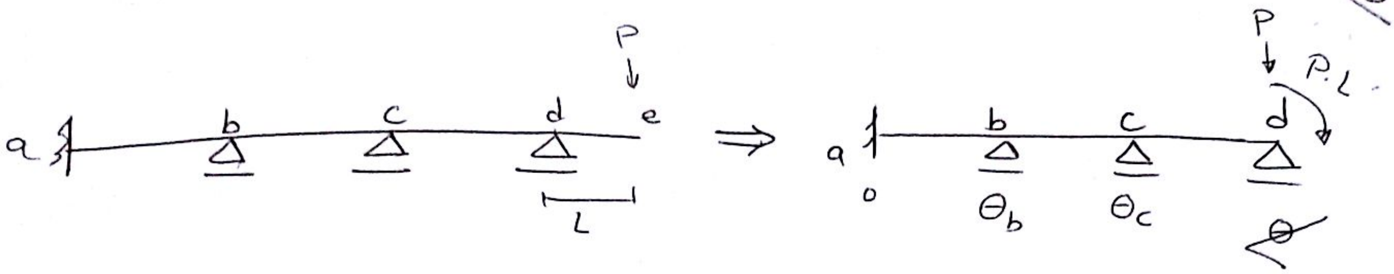
$D.o.f = \theta_b$

أما في حالة Un-symmetric (عدم التماثل) فيتم حساب كل

D.o.f

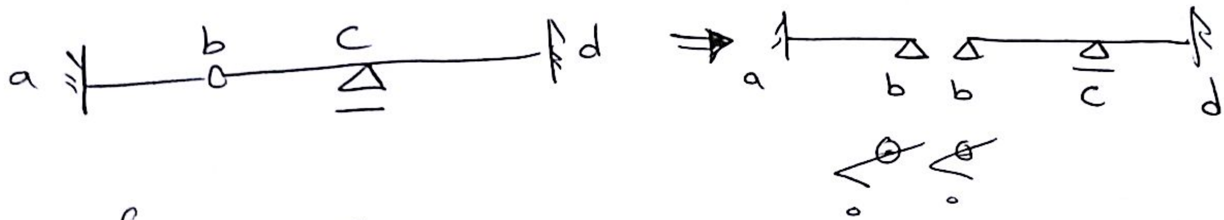
$D.o.f = \theta_b , \theta_c , \Delta_H$





$D.o.f = \theta_b , \theta_c$ نهمل اي θ
معلوم عندها
العزم سواء
برقم أو بصفر

$\Rightarrow D.o.f = \theta_b$



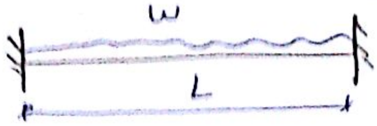
$D.o.f = \Delta_b , \theta_{b_r} , \theta_{b_L} ,$ $\Rightarrow$ $D.o.f = \Delta_b , \theta_c$
θ_c (عادى) (modified

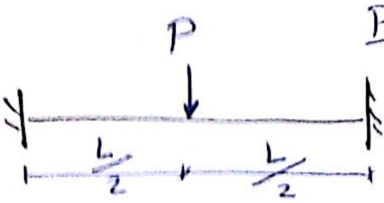
---

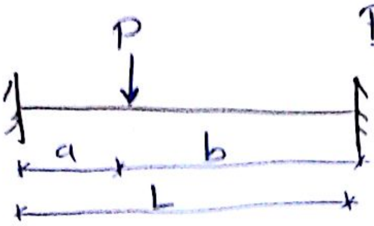
2 F.E.M

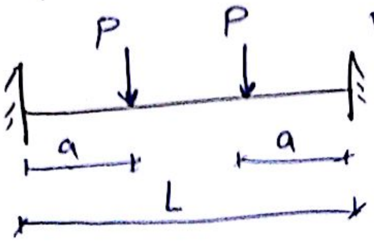
General	Modified

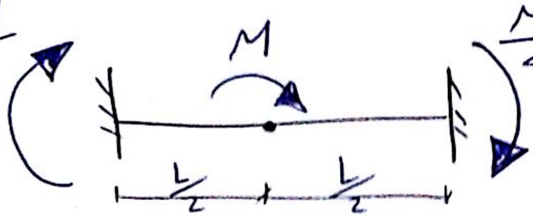
General

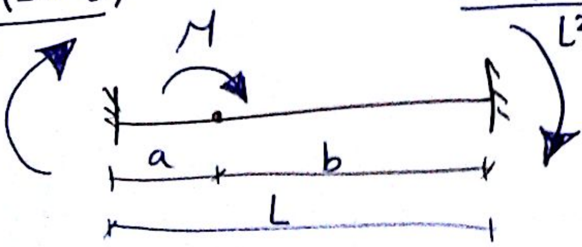
$\frac{wL^2}{12}$  $\frac{wL^2}{12}$

$\frac{PL}{8}$  $\frac{PL}{8}$

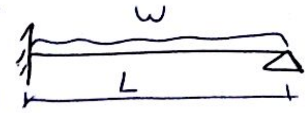
$\frac{Pab^2}{L^2}$  $\frac{Pba^2}{L^2}$

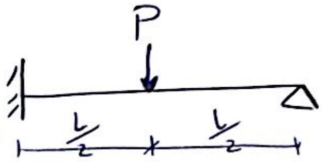
$\frac{Pa(L-a)}{L}$  $\frac{Pa(L-a)}{L}$

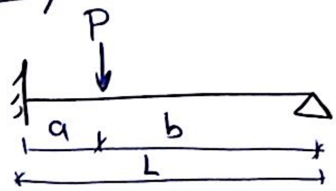
$\frac{M}{4}$  $\frac{M}{4}$

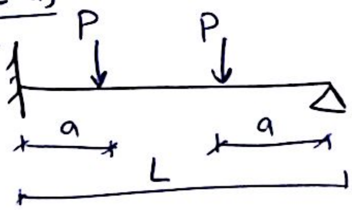
$\frac{Mb(2a-b)}{L^2}$  $\frac{Ma(2b-a)}{L^2}$

Modified

$\frac{wL^2}{8}$ 

$\frac{3PL}{16}$ 

$1.5\left(\frac{Pab^2}{L^2}\right)$ 

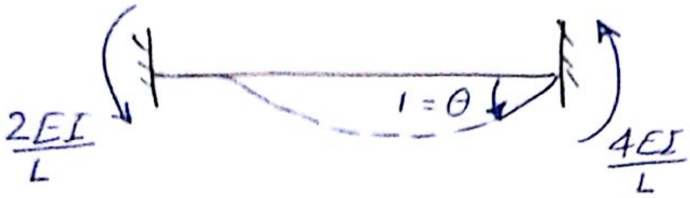
$1.5\,\frac{Pa(L-a)}{L}$ 

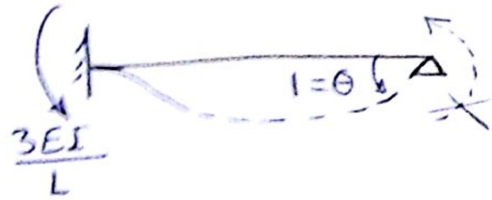
* يمكن حفظ قيم General فقط وللحصول على قيم modified يتم ضرب القيم في 1.5

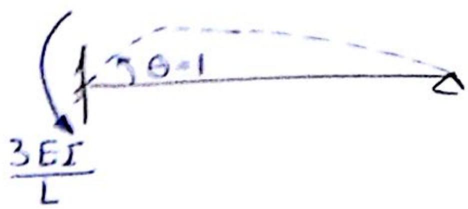
$$\frac{wl^2}{12} * 1.5 \Rightarrow \frac{wl^2}{8}$$

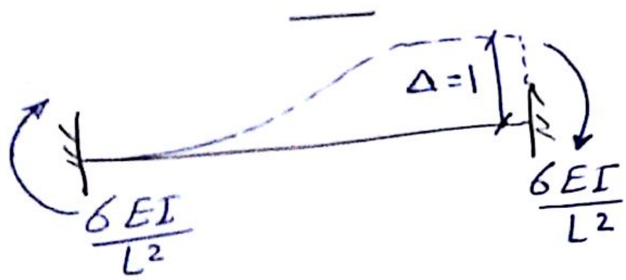
وذلك لاعتبار العزم يصفر عند △ فنرحل العزم ب

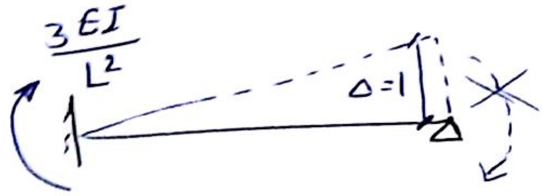
$$\frac{wl^2}{12} + \frac{1}{2}\frac{wl^2}{12}$$ (نصفه) C.O.m

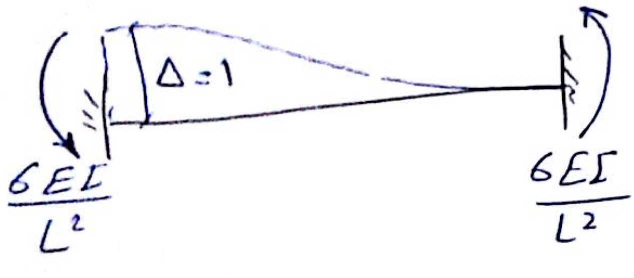


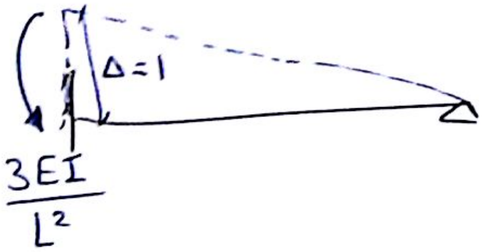




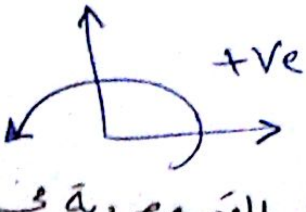








⇐ مع مراعاة قاعدة الإشارات حيث اتجاه C.W عقارب الساعة -ve و a.c.w بـ +ve



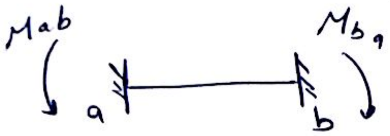
⇐ كل القيم معرفة في Δ أو θ ولكن هنا كانت بـ ①

1 SLOPE DEFLECTION

خطوات الحل :

1. D.o.f
2. F.E.M
3. Slope Deflection eq.

كتابة معادلاتها والتعويض بالموجود في المسألة .

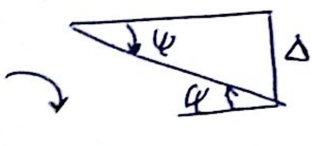


$$M_{ab} = M_{ab\,F.E.M} + \frac{EI}{L}\left(4\theta_a + 2\theta_b - \frac{6\Delta_{ab}}{L}\right)$$

$$M_{ba} = M_{ba\,F.E.M} + \frac{EI}{L}\left(4\theta_b + 2\theta_a - \frac{6\Delta_{ba}}{L}\right)$$

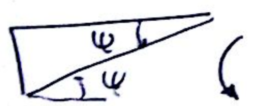
$\Delta_{ba} = \Delta_{ab} \Rightarrow$ فرق Δ بين النقطتين a, b $\Leftarrow$

وتحدد اشارتها حسب الزاوية ψ

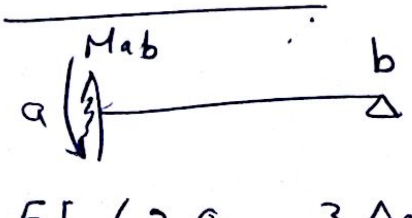


$\Delta = -ve$

$$\frac{\Delta}{L} = \psi$$



$\Delta = +ve$



Modif.

$$M_{ab} = M_{ab\,F.E.M} + \frac{EI}{L}\left(3\theta_b - \frac{3\Delta_{ab}}{L}\right)$$

$M_{ba} = \text{Zero}$

Scanned by CamScanner

من هذه الخطوة ينتج عندك معادلات للعزوم المطلوبة
ولكنه مجهولة في D.O.F الموجودة

④ Equilibrium:

* يجب عمل اتزان لايجاد المجاهيل D.O.F

$\Rightarrow \Sigma M$ @ any point $= 0 \Rightarrow$ في حالة المجهول θ

$\Rightarrow \Sigma F = 0 \Rightarrow$ في حالة المجهول Δ

⑤ Back sub. in step ③:

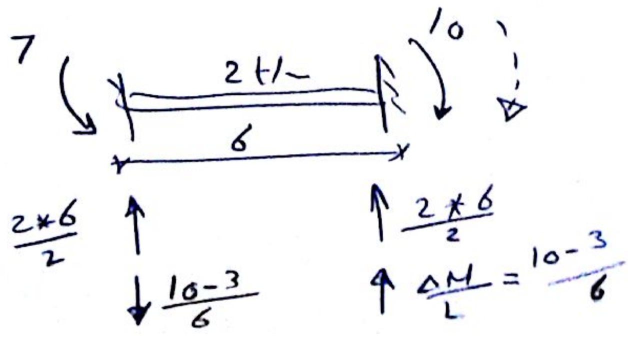
نرجع نعوض في الخطوة ③ نحصل على قيم العزوم

⑥ Free Body Dia. نرسم

عبارة عن رسمة الشكل ولكن متقطع

ونجيب الـ Shear عن طريق الأحمال وعن طريق العزوم

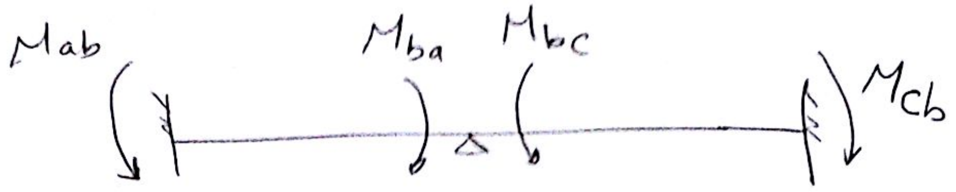
EX:

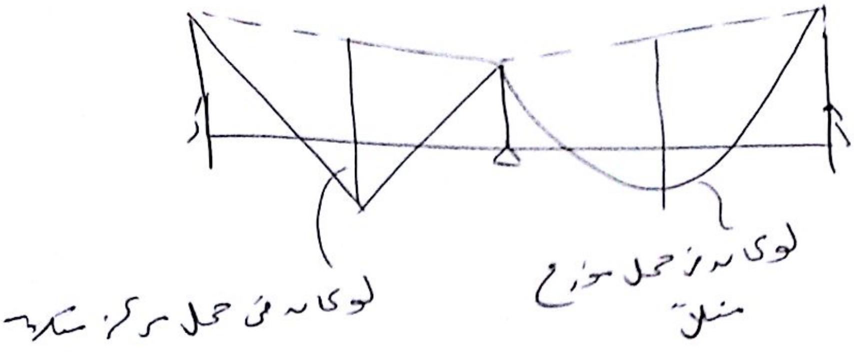


ثم نجمع R ونرسم shear

7 Draw B.M.D & S.F.D

نرسم بالقيم اللى حصلنا عليها وحسب الاشارات

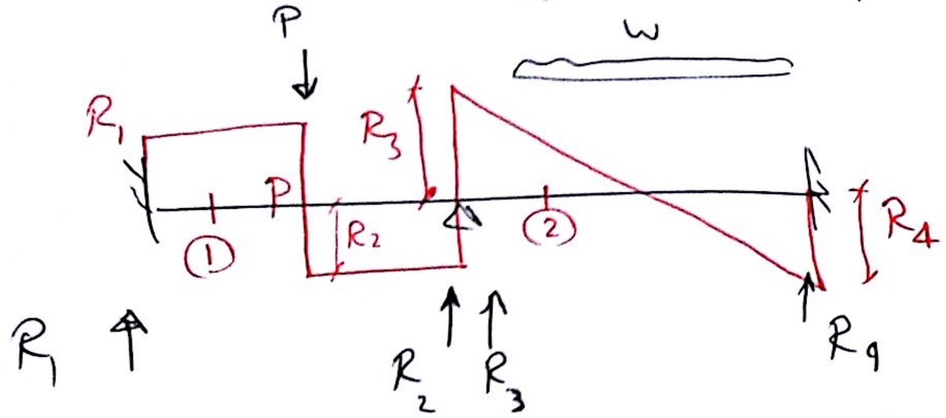




B.M.D

والـ shear حسب الرسم فى

(Free Body Diaghram)



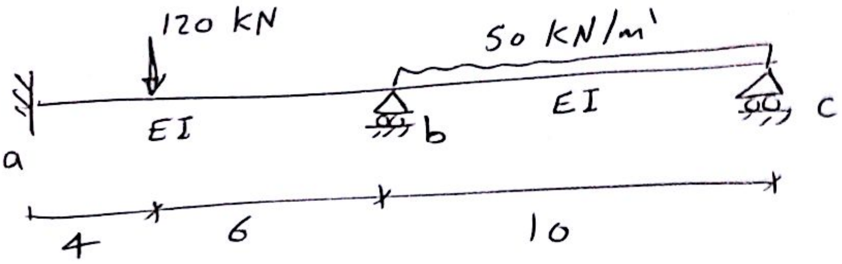
S.F.D

① → ثابت لأنه الأحمال الرأسية لم تتغير على طول هذا العنصر

② → يتغير لأنه الأحمال الرأسية تتغير على طول العنصر

EX

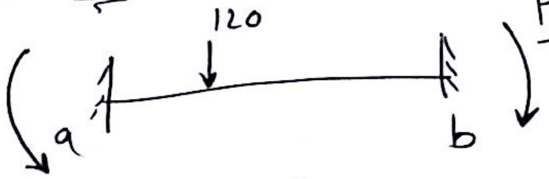
Draw B.M.D & S.F.D



Sol.

① D.o.f $\Rightarrow$ θ_b (modified)

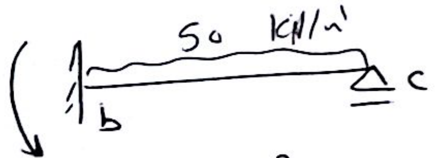
② F.E.M



$$\frac{Pba^2}{L^2} = \frac{120*6*4^2}{10^2} = \ominus 115.2 \text{ KN.m}$$

علشان
مع عقارب الساعة

$$\frac{Pab^2}{L^2} = \frac{120*4*6^2}{10^2} = +172.8 \text{ KN.m}$$



$$\frac{Wl^2}{8} = \frac{50*10^2}{8} = +625 \text{ KN.m}$$

③ Slope Defl. eq.

$$M_{ab} = M_{ab_{F.EM}} + \frac{EI}{L}\left(4\cancel{\theta_a} + 2\theta_b - \cancel{\frac{6\Delta}{L}}\right)$$

0   مجهولة   مفيش Δ

علشان $\dashv$ لا يوجد هبوط

$$\therefore M_{ab} = +172.8 + \frac{EI}{10}(2\theta_b) \longrightarrow ①$$

$$* \; M_{ba} = M_{ba_{F.E.M}} + \frac{EI}{L}\left(4\theta_b + 2\cancel{\theta_a}_{\,0} - \frac{6\cancel{\Delta}}{L}_{\,0}\right)$$

$$M_{ba} = -115.2 + \frac{EI}{10}(4\theta_b) \longrightarrow (2)$$

$$* \; M_{bc} = M_{bc_{F.E.M}} + \frac{EI}{L}\left(3\theta_b - \frac{3\cancel{\Delta}}{L}_{\,0}\right) \qquad \underline{\text{mod.}}$$

$$M_{bc} = +625 + \frac{EI}{10}(3\theta_b) \longrightarrow (3)$$

$$* \; M_{cb} = \text{Zero} \longrightarrow \Delta$$

---

(4) Equilibrium

$$\Sigma M @ b = 0 \Rightarrow M_{ba} + M_{bc} = 0$$

(*) لو كان فيه عزم مركز هيخش معايا و باشارته

$$eq(2) + eq(3) = 0$$

$$\left[-115.2 + \frac{EI}{10}(4\theta_b)\right] + \left[625 + \frac{EI}{10}(3\theta_b)\right] = 0$$

$$\Rightarrow \theta_b = \frac{-728.23}{EI}$$

(5) Back Sub. in step (3)

تعويض في المعادلات

$$* M_{ab} = 172.8 + \frac{EI}{10}\left(2 * \frac{-728.23}{EI}\right) = +27.14 \text{ KN.m}$$

$$* M_{ba} = -115.2 + \frac{EI}{10}\left(4 * \frac{-728.23}{EI}\right) = -406.5 \text{ KN.m}$$

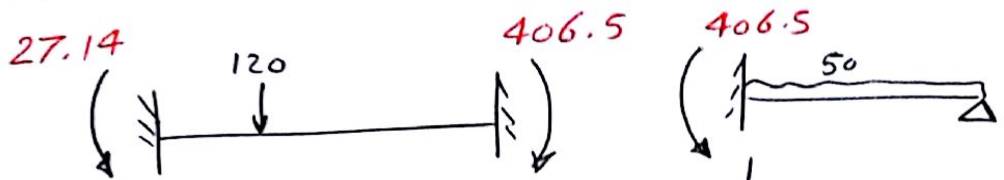
$$* M_{bc} = 625 + \frac{EI}{10}\left(3 * \frac{-728.23}{EI}\right) = +406.5 \text{ KN.m}$$

لازم العزم عند نفس النقطة يكون نفس القيمة لو

مكنش فيه عزم مركز يعني $M_{ba} = M_{bc}$

---

(6) Free Body Dia.



(R) From Load moment

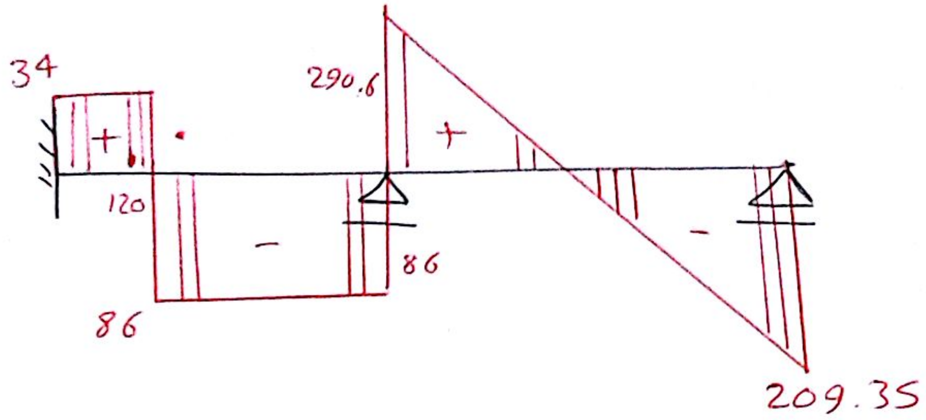
$\uparrow \frac{120 * 6}{10} = 72$ | $\uparrow \frac{120 * 4}{10} = 48$ | $\uparrow 250$ | $\uparrow 250$

$\downarrow 38$ | $\uparrow \frac{406.5 - 27.14}{10} \simeq 38$ | $\uparrow 40.65$ | $\downarrow \frac{406.5}{10} = 40.65$

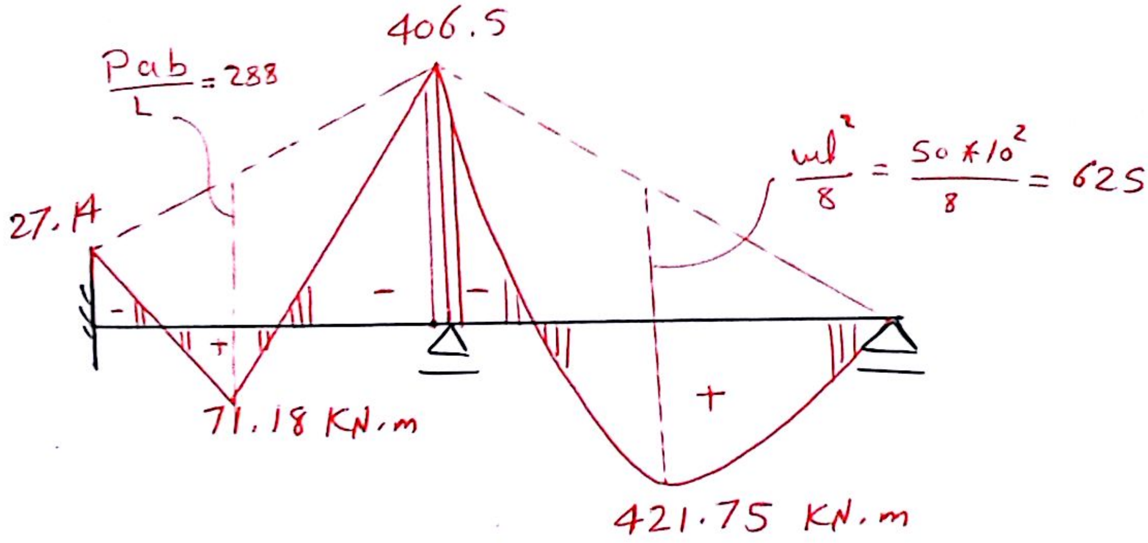
ΣR $\uparrow 34$ | $\uparrow 86$ | $\uparrow 290.65$ | $\uparrow 209.35$

⑦ B.M.D & S.F.D

S.F.D

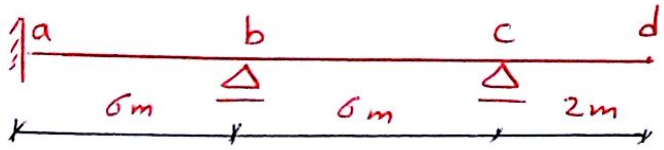


B.M.D



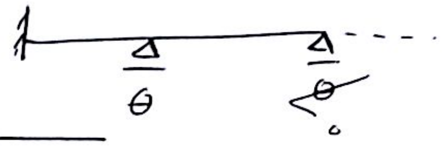
Ex

Draw B.M.D & S.F.D due to sett. at "a" = 0.2 Cm and rotation at "a" = 0.005 C.W, sett. at "b" = 1 Cm and sett. at "c" = 0.5 Cm. ($EI = 10000\ m^2.t$)



Sol.

① $D.o.f = \theta_b$ (modi.)



② F.E.M:

No Load → F.E.M = Zero

③ Slope Deflection eq:

نلاحظ هنا أن هناك Δ , θ معطى فيجب التعويض بهم.

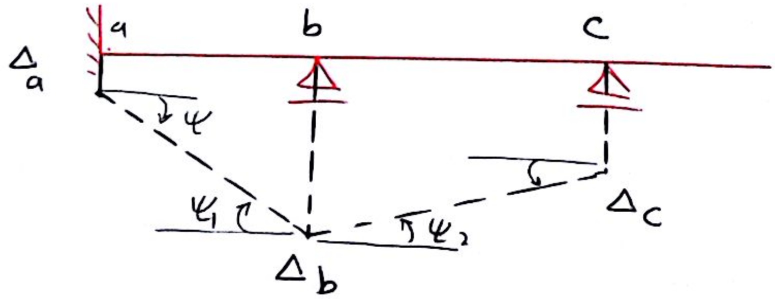
given:

$\Delta_a = 0.2\ Cm$

$\Delta_b = 1\ Cm$

$\Delta_c = 0.5\ Cm$

$\theta_a = 0.005\ C.W$



$\Delta_{ab} = \Delta_{ba} = -ve$ | $\Delta_{bc} = \Delta_{cb} = +ve$

$= -(1 - 0.2) = -0.8\ Cm$ | $= 1 - 0.5 = 0.5\ Cm$

$\psi_1 = \frac{\Delta}{L} \Rightarrow \psi = \frac{-0.8}{6} * \frac{1}{100}$ | $\psi_2 = \frac{0.5}{6} * \frac{1}{100}$

التحويل لمتر

لا يمكن التعويض بالمجاهيل

$$* \; M_{ab} = \underbrace{M_{ab\,F.E.M}}_{0} + \frac{EI}{L}\left(4\theta_a + 2\theta_b - 6\frac{\Delta_{ab}}{L}\right)$$

معطى   معطى

$$M_{ab} = \frac{EI}{6}\left(4(-0.005) + 2\theta_b - \frac{6\left(\frac{-0.8}{100}\right)}{6}\right)$$

C.W

تعويض بالإشارة

$$M_{ab} = \frac{EI}{6}\left(2\theta_b - 0.012\right) \rightarrow (1)$$

$$* \; M_{ba} = \underbrace{M_{ba\,F.E.M}}_{0} + \frac{EI}{L}\left(4\theta_b + 2\theta_a - \frac{6\Delta_{ba}}{L}\right)$$

معطى   معطى

$$M_{ba} = \frac{EI}{6}\left(4\theta_b + 2(-0.005) - \frac{6\left(\frac{-0.8}{100}\right)}{6}\right)$$

$$M_{ba} = \frac{EI}{6}\left(4\theta_b - \frac{1}{500}\right) \rightarrow (2)$$

modi.

$$* \; M_{bc} = \underbrace{M_{bc\,F.E.M}}_{0} + \frac{EI}{L}\left(3\theta_b - 3\frac{\Delta_{bc}}{L}\right)$$

معطى

$$M_{bc} = \frac{EI}{\cancel{6}_{\,2}}\left(\cancel{3}\theta_b - \frac{\cancel{3}\left(\frac{0.5}{100}\right)}{6}\right)$$

$$= \frac{EI}{2}\left(\theta_b - \frac{1}{1200}\right) \rightarrow (3)$$

$$* \; M_{cb} = \text{Zero}$$

---

(4) Equilibrium:

$$\Sigma M@b = 0 \Rightarrow M_{ba} + M_{bc} = 0$$

$$eq(2) + eq(3) = 0$$

$$\frac{EI}{6}\left(4\theta_b - \frac{1}{500}\right) + \frac{EI}{2}\left(\theta_b - \frac{1}{1200}\right) = 0$$

$$\therefore \theta_b = 6.43 * 10^{-4} \text{ rad}$$

(5) Sub.

$$M_{ab} = \frac{EI}{6}\left(2\theta_b - 0.012\right)$$

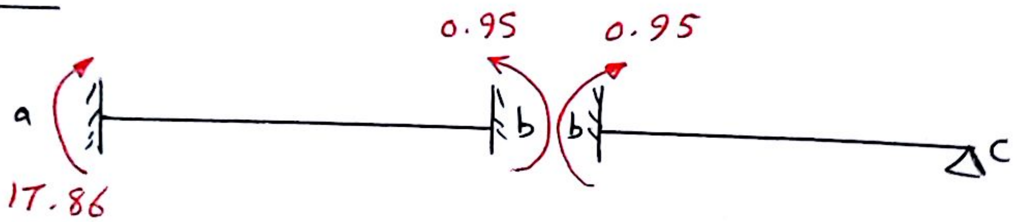
$$= \frac{10000}{6}\left(2 * 6.43 * 10^{-4} - 0.012\right) = -17.86 \text{ m.t}$$

$$M_{ba} = \frac{10000}{6}\left(4 * 6.43 * 10^{-4} - \frac{1}{500}\right) = +0.95 \text{ m.t}$$

$$M_{bc} = \frac{10000}{2}\left(6.43 * 10^{-4} - \frac{1}{1200}\right) = -0.95 \text{ m.t}$$

(6) B.M.D & S.F.D

Free Body Dia.



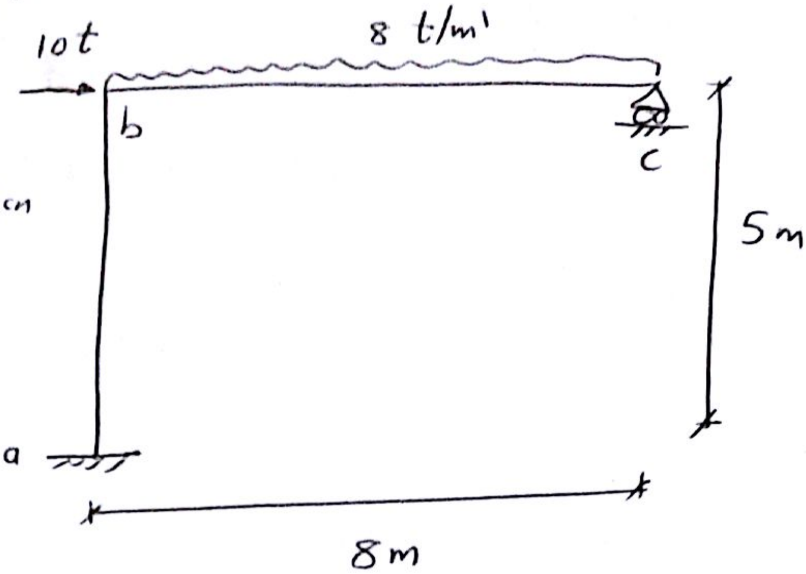
R → load = 0

→ moment ⇒ ↓ $\frac{17.86 - 0.95}{6} = 2.82$ ↑ 2.82 ↓ $\frac{0.95}{6} = 0.16$ ↑ 0.16

↓ 2.82 ↑ 2.66 ↑ 0.16

EX: "Frame with Sway"

Draw B.M.D & S.F.D by using slope deflection Method ...



Sol.

① **D.o.f**

@ a = 0

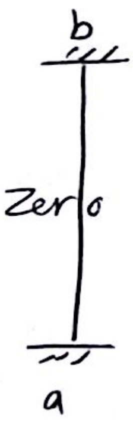
@ b = θ_b , Δ_x

@ c = $\cancel{\theta_c}$, Δx

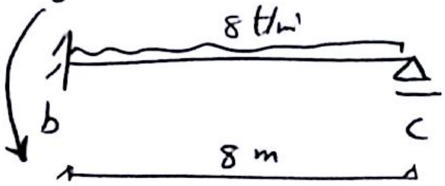
neglect → علشان ~ عارفين العزم عندها

$$\Rightarrow \boxed{D.o.f = \theta_b \, , \, \Delta_x}$$

② **F.E.M**



$\frac{wl^2}{8} = 64$ m.t



③ Slope deflection eq. $\psi_{ab} = \frac{\Delta x}{L} \Rightarrow \boxed{\Delta ab = \Delta ba = \Delta_x}$

$$\Rightarrow M_{ab} = M_{ab_{F.E.M}} + \frac{EI}{L}\left(4\theta_a + 2\theta_b - \frac{6\Delta ab}{L}\right)$$

$$M_{ab} = \frac{EI}{5}\left(2\theta_b - \frac{6\Delta ab}{5}\right)$$

$$\Rightarrow M_{ba} = M_{ba_{F.E.M}} + \frac{EI}{L}\left(4\theta_b + 2\theta_a - \frac{6\Delta_{ba}}{L}\right)$$

$$M_{ba} = \frac{EI}{5}\left(4\theta_b - \frac{6\Delta_{ba}}{5}\right)$$

$$\Rightarrow M_{bc} = M_{bc_{F.E.M}} + \frac{EI}{L}\left(3\theta_b - \frac{3\Delta_{bc}}{L}\right)$$

لا نأخذ Δ إلا التي تؤثر عمودي على الحركة ومن الت
تسبب زاوية ψ

$$M_{bc} = 64 + \frac{EI}{8}(3\theta_b)$$

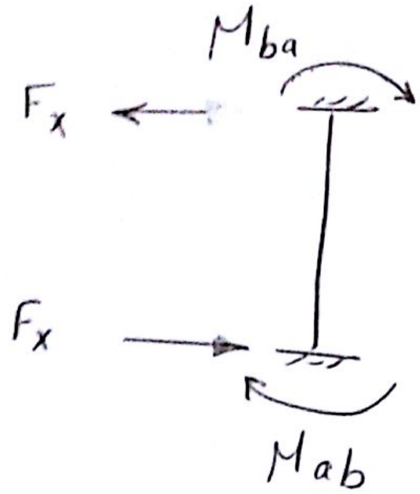
---

④ equill.

@ Joint ⓑ $\Rightarrow M_{ba} + M_{bc} = 0$

$$\left[\frac{EI}{5}\left(4\theta_b - \frac{6\Delta}{5}\right)\right] + \left[64 + \frac{EI}{8}(3\theta_b)\right] = 0 \longrightarrow ①$$

معادلة في مجهولين ← لازم نجيب معادلة
ثانية ← هنجيبها من **Shear**

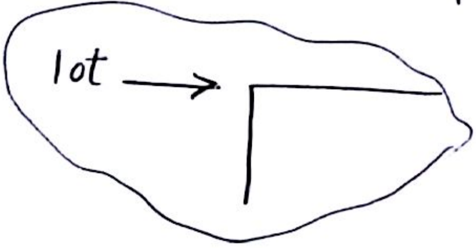


$$F_x = \frac{\Delta M}{L} = \frac{M_{ab} + M_{ba}}{L}$$ نعوض بالمعادلات

$$F_x = \frac{\left[\frac{EI}{5}\left(2\theta_b - \frac{6\Delta}{5}\right)\right] + \left[\frac{EI}{5}\left(4\theta_b - \frac{6\Delta}{5}\right)\right]}{5}$$

$\sum F_x = 0$ باتزان القوى في اتجاه (X)

$$F_x = 10\,t$$



$$-50 = \left[\frac{EI}{5}\left(2\theta_b - \frac{6\Delta}{5}\right)\right] + \left[\frac{EI}{5}\left(4\theta_b - \frac{6\Delta}{5}\right)\right] \longrightarrow (2)$$

وديه هي المعادلة الثانية بحل المعادلتين

نحصل على θ_b , Δ

(4) back. Sub. in step (3)

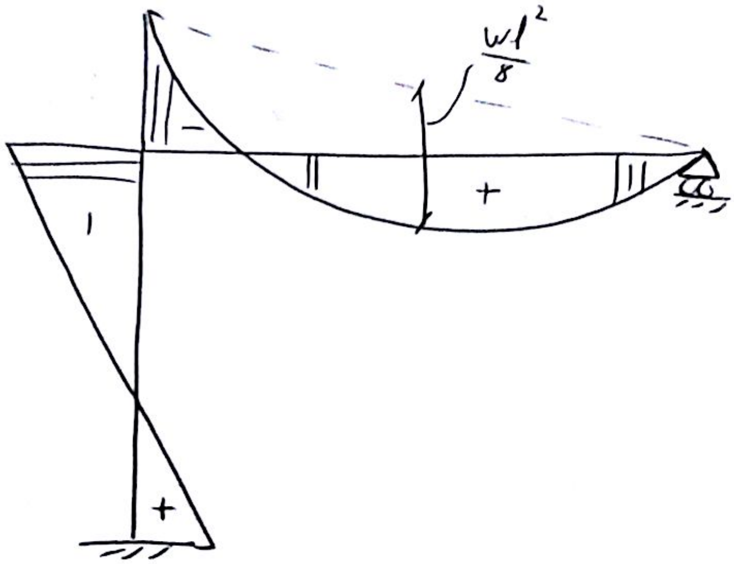
نرجع نعوض بقيم Δ ، θ_b في المعادلات نحصل على

$M_{ab} =$

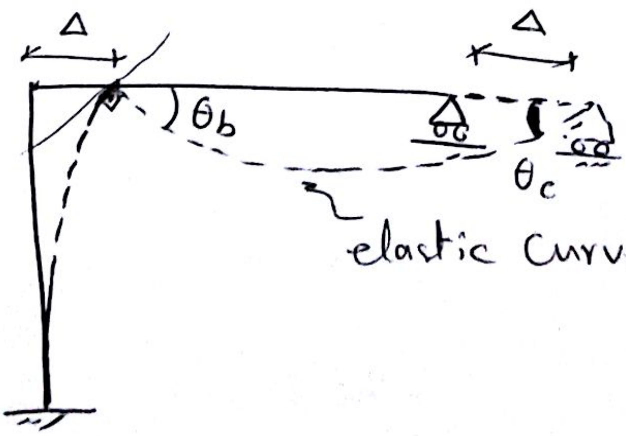
$M_{ba} =$

$M_{bc} =$

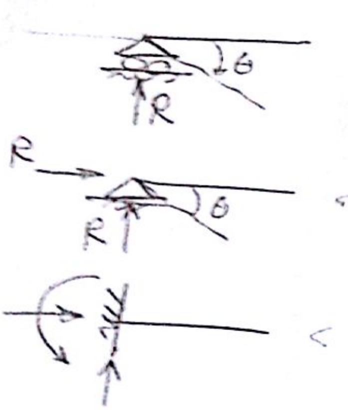
B.M.D



elastic Curve
Deformation



Revision on Deflection & elastic curve



$D.o.f = \Delta_x, \theta$

$R = ①$ ممنوعة Δy

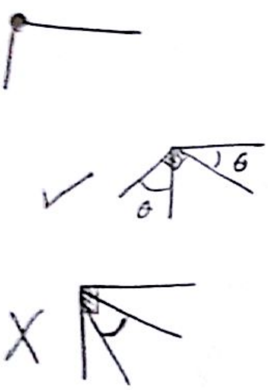
$D.o.f = \theta$

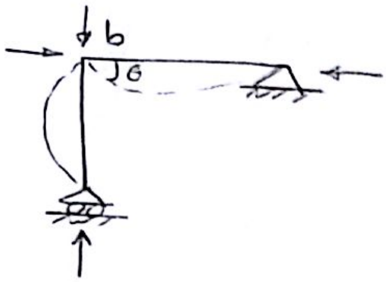
$R = ②$

$D.o.f = 0$

$R = ③$

لكل نقطة
حركات متاحة
وحركات ممنوعة
بسبب ردود الأفعال



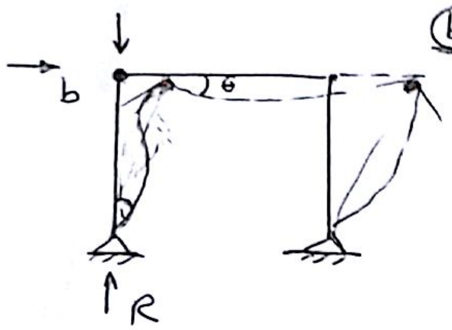


<u>ⓑ</u>

$D.o.f = \theta$

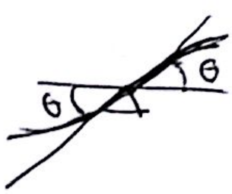
$R = 2$

لازم هنا
نشوف اتصالها



ⓑ $D.o.f = \Delta_x, \theta$

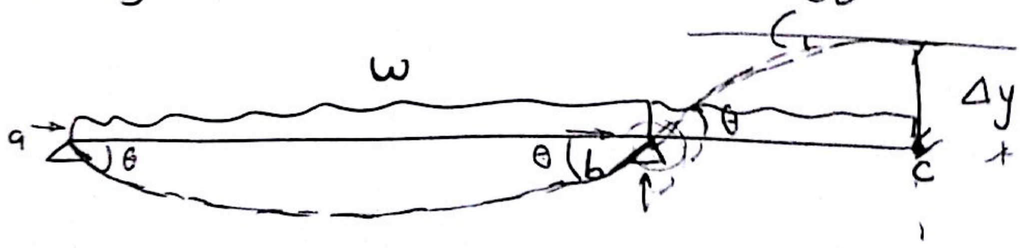
$R = ①$



ⓐ $D.o.f = \theta_a$

ⓑ $D.o.f = \theta_b$

Deformation = elastic curve

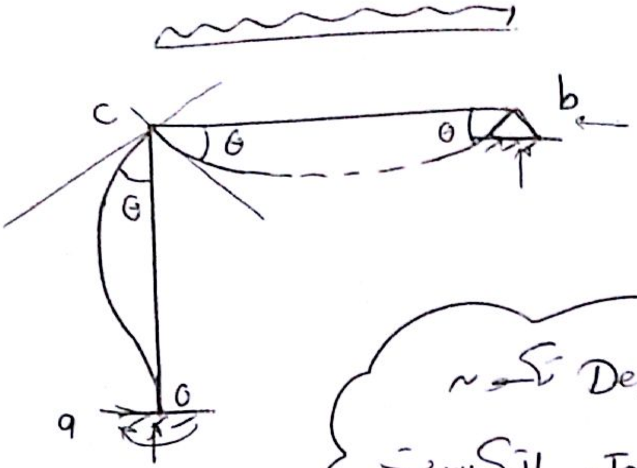


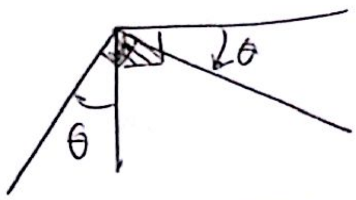
ⓐ ⓑ

ⓑ ⓑ

ⓒ, $\boxed{\Delta y, \theta_c}$

$\Delta y, \theta_c$





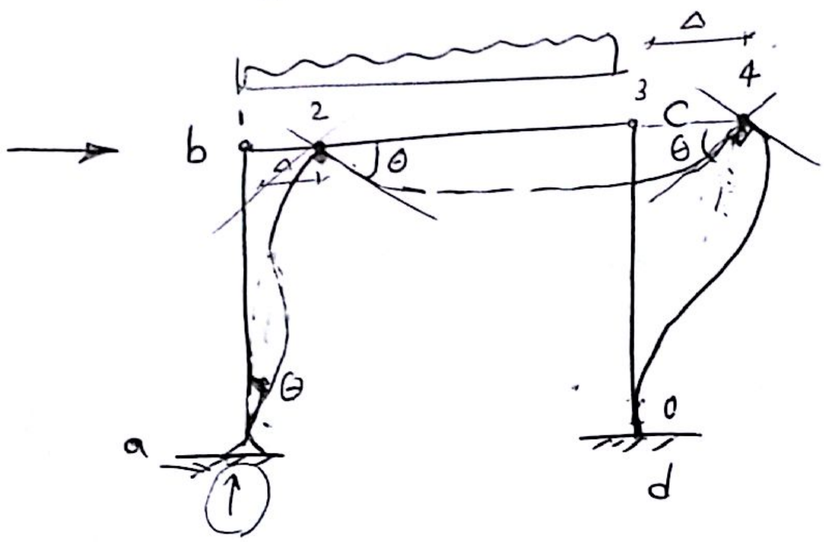
لازم الزاوية قبل Deformation تكون هي بعده وإلا تكون الـ Joint اتكسرت

D.o.f

ⓐ $= 0$

ⓑ θ_b

ⓒ θ_c



D.o.f

ⓐ $= \theta_a$

ⓑ $= \Delta_x , \theta_b$

ⓒ $= \Delta_x , \theta_c$

ⓓ $= 0$

الخلاصة علشان ترسم صح :

(١) حدد الحركات المسموح بها جنب كل نقطة D.o.f [درجات الحرية].

(٢) ابدأ اسمح بالحركات طبقاً للأحمال الموجودة مع مراعاة الزاوية قبل الـ Deformation هي نفسها بعد الـ Deformation

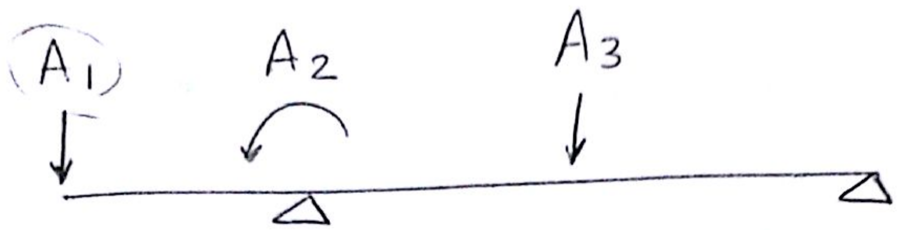
Structure Analysis

تصنيف طرق التحليل

الى

Force Method	Displacement
Virtual Work	Slope deflection
flexability	Stiffness

EX:



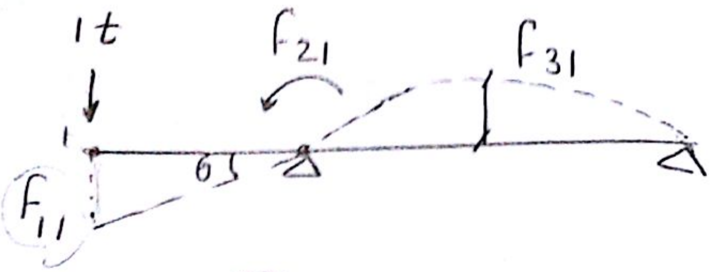
Given: A_1 , A_2 , A_3 $\xrightarrow{\text{are}}$ D.o.f

Physical meaning of flexability & stiffness Method

⇐ مطلوب اعطاء المعنى لكل من الطريقتين flexability و stiffness عن طريق الرسم

flexability	stiffness
↓	↓
نفرض Force	نفرض Deformation $(\Delta \& \theta)$
↓	↓
نرسم Deformation	نرسم نتيجة الفرض فقط
↓	↓
تحسب المجاهيل ثم القوى الداخلية	تحسب المجاهيل ثم القوى الداخلية

Flexability



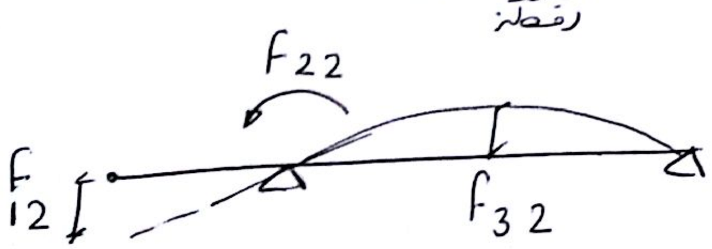
$A_1 = 1\,t$

f_{11}

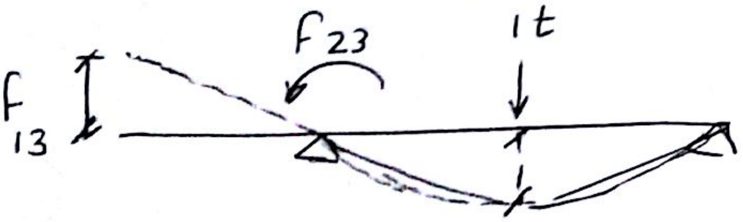
Displacement @ ① point $\xrightarrow{\text{لما}}$ ① احمل كام عند نقطة

f_{21}

Displacement @ ② $\xrightarrow{\text{لما}}$ ① احمل عند نقطة

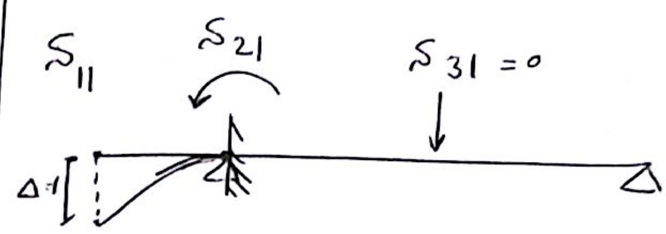


$A_2 = 1\,m.t$

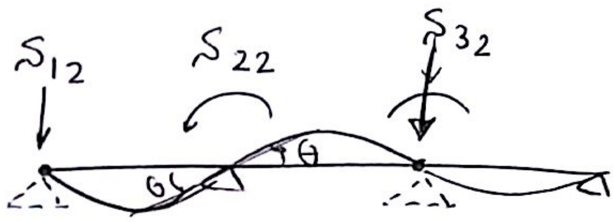


$A_3 = 1\,t$ $\quad f_{33}$

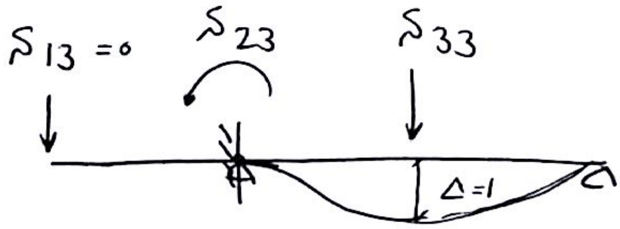
Stiffness



$A_1 = \Delta = 1$



$A_2 = \theta = 1$



$A_3 = \Delta = 1$

نفرض Deformation معين
ونمنع باقي D.o.f
من الحركة للحصول على
stiffness أو المقاومة التي
ستقاوم هذه الحركة.

Flexability Matrix

$$\begin{array}{cccc} & \text{Mode ①} & \text{Mode ②} & \text{Mode ③} \\ f = & \multicolumn{3}{c}{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}} \end{array}$$

$$\delta_{10} + (\delta_{11})(X_1) + (\delta_{21})(X_2) = \delta_{act.}$$

Virtual work eq.

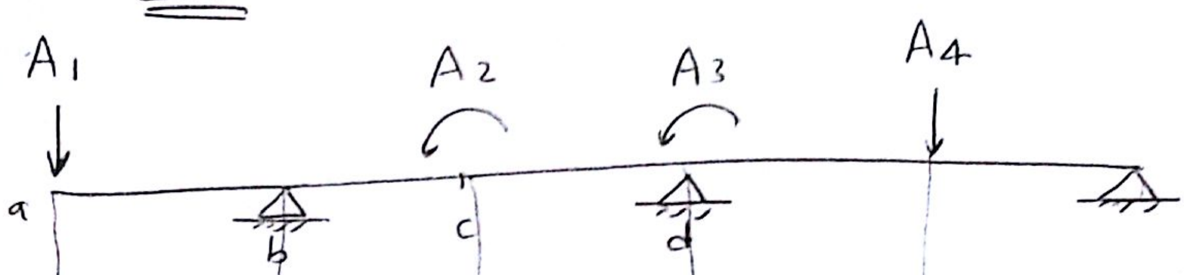
ترجمة المعف على شكل
مصفوفة تدخل في
الحل بعد ذلك
لإيجاد المجاهيل (θ, Δ) ..
ثم القوى الداخلية .

* وهنا نكتبها في Matrix
على عكس طريقة Virtual Work
سابقاً لأن المجاهيل هنا
كثيرة ليس إلا .

Stiffness Matrix

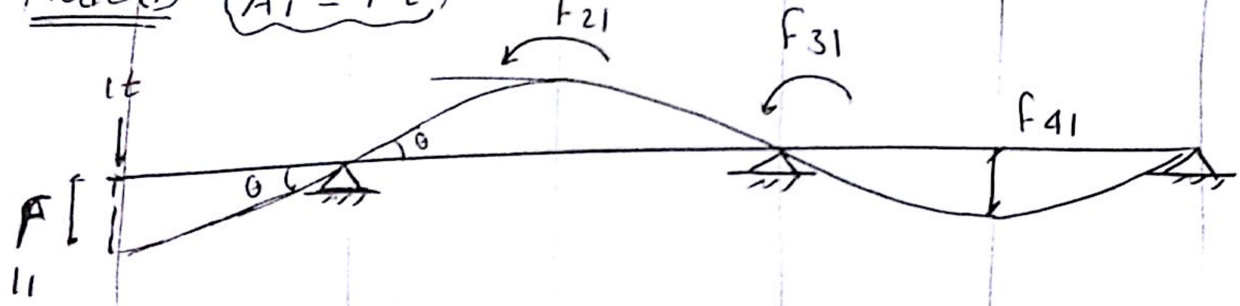
$$\begin{array}{cccc} & \text{Mode ①} & \text{Mode ②} & \text{Model} \\ S = & \multicolumn{3}{c}{\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}} \end{array}$$

Ex:

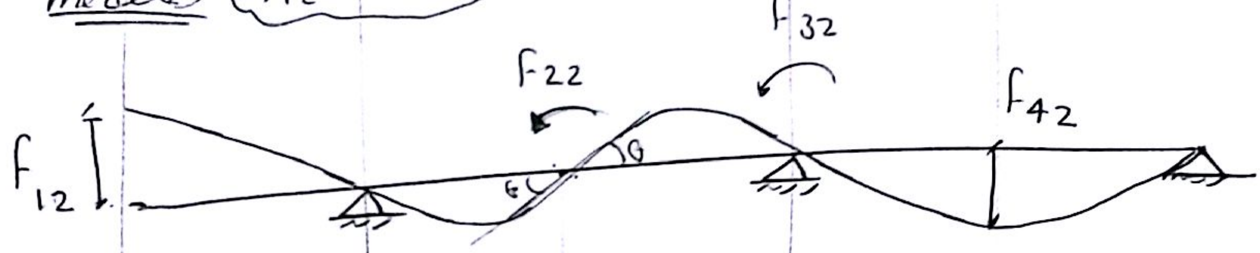


Physical meaning of flexability Method

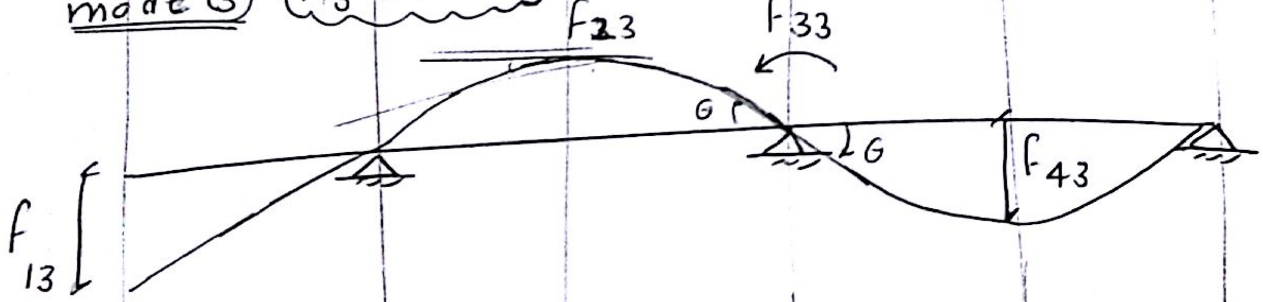
Mode ① ($A_1 = 1\ t$)



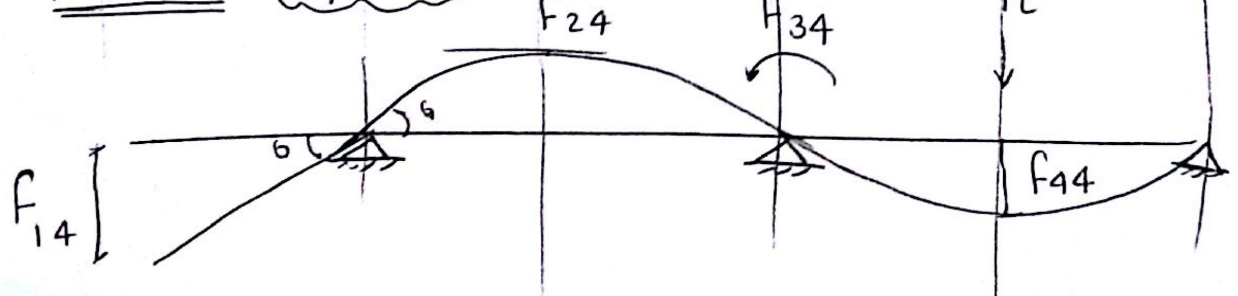
mode ② ($A_2 = 1\ m.t$)



mode ③ ($A_3 = 1\ m.t$)



mode ④ ($A_4 = 1\ t$)



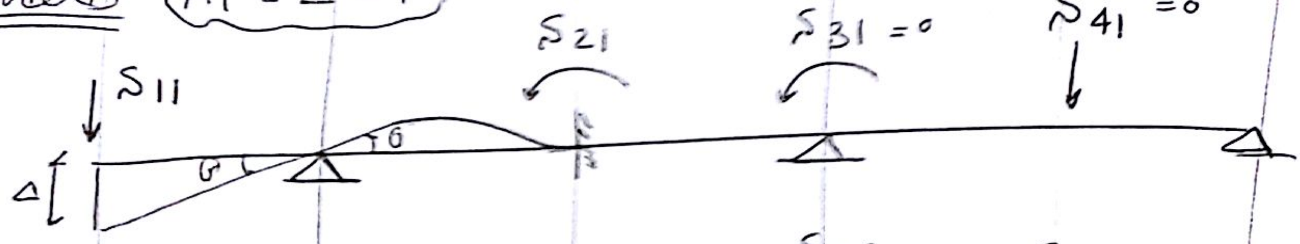
Flexability Matrix:

$$
\begin{array}{ccccc}
 & \text{Node ①} & \text{Node ②} & \text{Node ③} & \text{Node ④} \\
f = & \left[\begin{matrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{41} \end{matrix} \right. & \begin{matrix} f_{12} \\ f_{22} \\ f_{32} \\ f_{42} \end{matrix} & \begin{matrix} f_{13} \\ f_{23} \\ f_{33} \\ f_{43} \end{matrix} & \left. \begin{matrix} f_{14} \\ f_{24} \\ f_{34} \\ f_{44} \end{matrix} \right]
\end{array}
$$

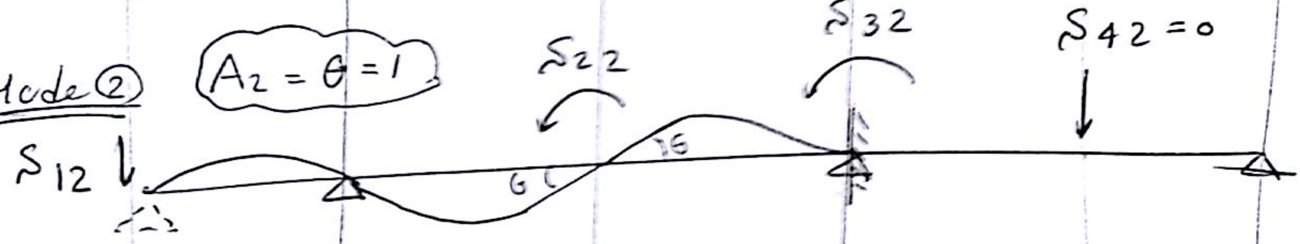
Ex:

Physical Meaning of Stiffness Method

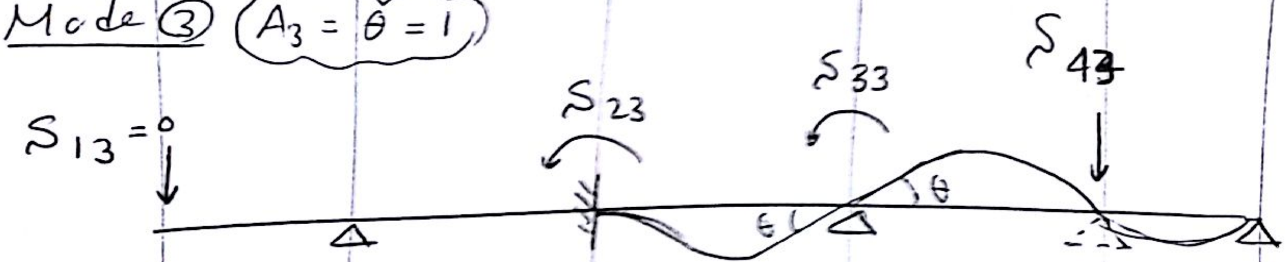
Mode ① ($A_1 = \Delta = 1$)



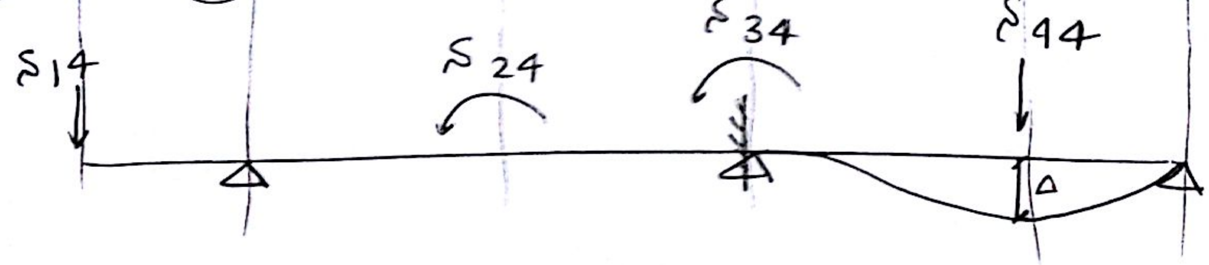
Mode ② ($A_2 = \theta = 1$)



Mode ③ ($A_3 = \theta = 1$)



Mode ④ ($A_4 = \Delta = 1$)



$$
S = \begin{array}{cccc}
\text{Mode } 1 & \text{Mode } 2 & \text{Mode } 3 & \text{Mode } 4 \\
\end{array}
$$

$$
S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21} & S_{22} & S_{23} & S_{24} \\
S_{31} & S_{32} & S_{33} & S_{34} \\
S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}
$$

* للسهولة بعد الفهم :

$$
\begin{bmatrix}
S_1 & S_1 & S_1 \\
S_2 & S_2 & S_2 \\
S_3 & S_3 & S_3 \\
\vdots & \vdots & \vdots
\end{bmatrix} \Rightarrow
$$

ثم عند كل Mode

ضع رقمه

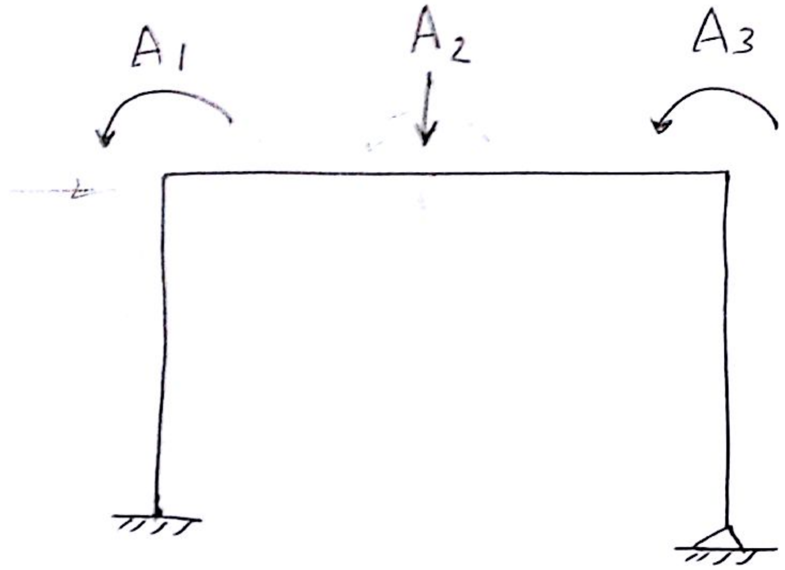
S_{11}

S_{21}

S_{31}

$\vdots$

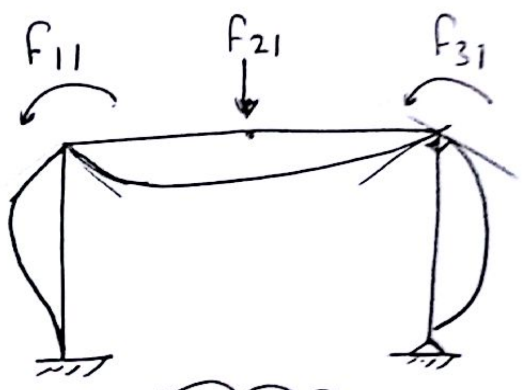
Ex: Get the Physicall meaning of flexability & stiffness Method....



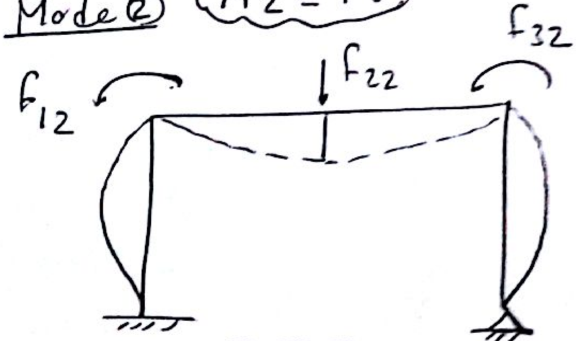
Flexability	Stiffness

Flexability

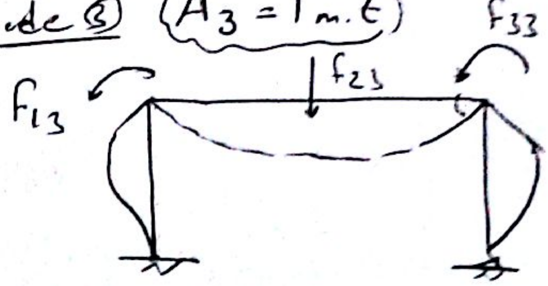
Mode ① $A_1 = 1\,m.t$



Mode ② $A_2 = 1\,t$

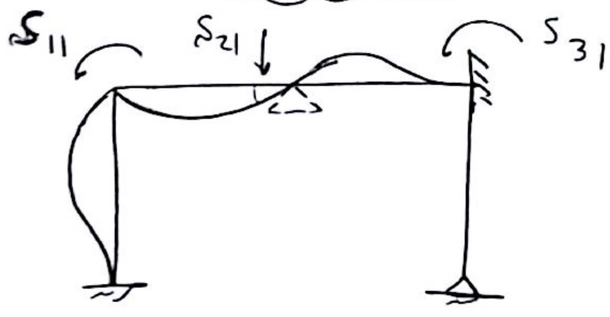


Mode ③ $A_3 = 1\,m.t$

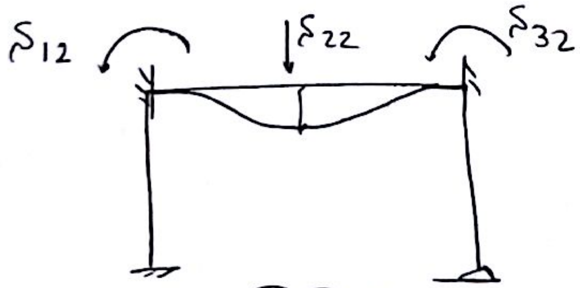


Stiffness

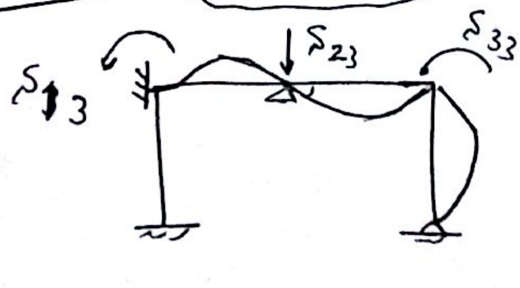
Mode ① $A_1 = \theta = 1$



Mode ② $A_2 = \Delta = 1$



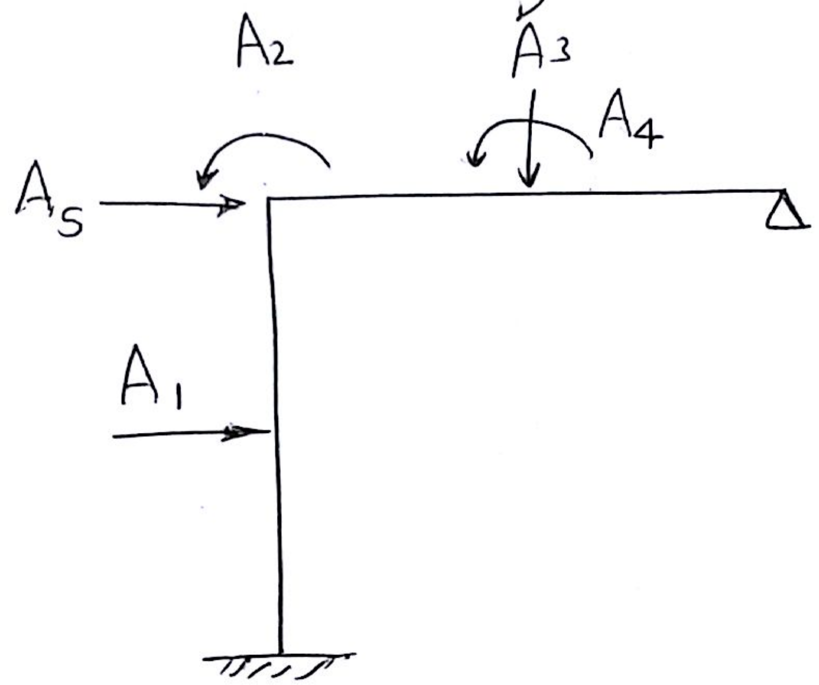
Mode ③ $A_3 = \theta = 1$



$$f = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

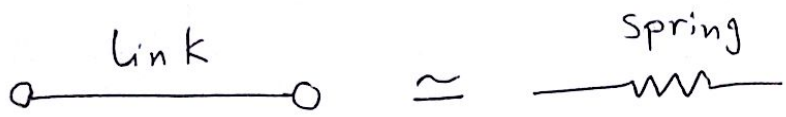
H.W

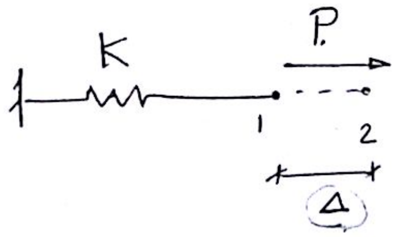


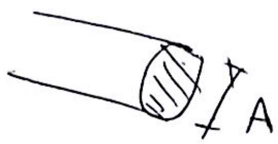
Get Physical meaning for flexibility & Stiffness Method by the given Degree of freedom.

Direct stiffness Method

① Truss







$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{P}{A} \quad , \quad \text{Strain} = \frac{\Delta}{L}$$

Δ الاستطالة

L الطول الأصلي

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\Delta/L}$$

$$P = E\,\frac{\Delta}{L}\cdot A \Rightarrow \boxed{P} = \overbrace{\boxed{\frac{EA}{L}}}^{K}_{\text{Stiffness}} \boxed{\Delta}$$

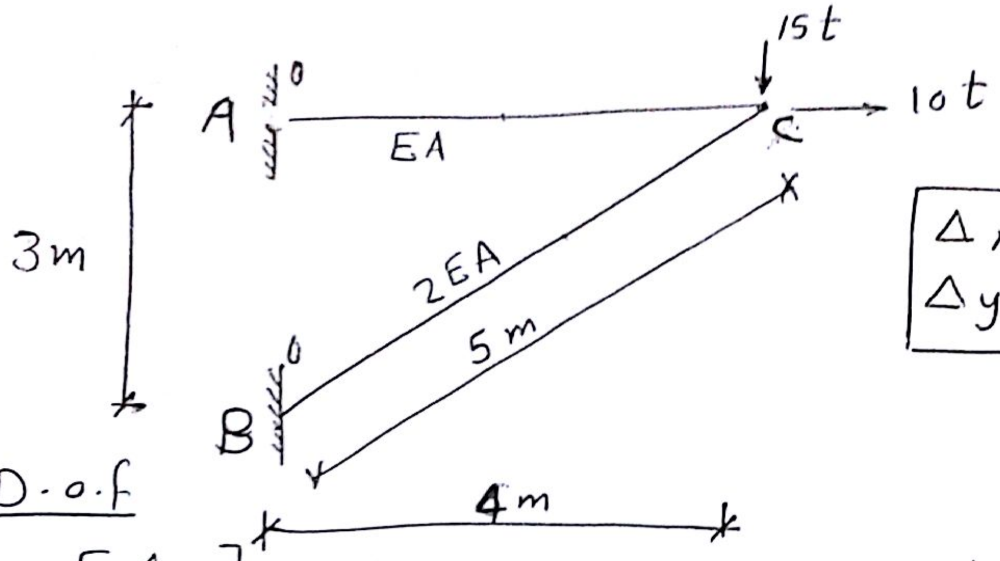
$$P = K \cdot \Delta$$

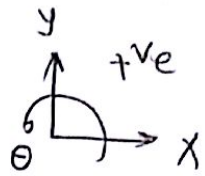
$$\begin{bmatrix} \text{Load}_1 \\ \text{Load}_2 \end{bmatrix} = \begin{bmatrix} s & s \\ s & s \end{bmatrix} \begin{bmatrix} \Delta \\ \Delta \end{bmatrix}$$

$$k = S$$

$$k = [\quad]$$

EX: Calculate the displacement at $\boxed{C}$ & the Forces in member (AB & BC)





$$\boxed{\begin{matrix}\Delta x \\ \Delta y\end{matrix}}$$

① D.o.f

$$D = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Sol.

$$P = K\,\Delta \Rightarrow [P] = [S]\,[D]$$

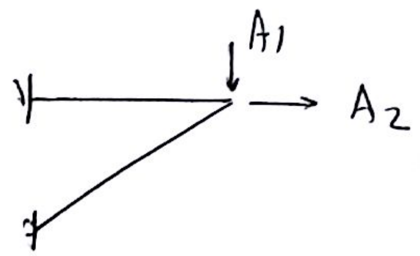
load vector

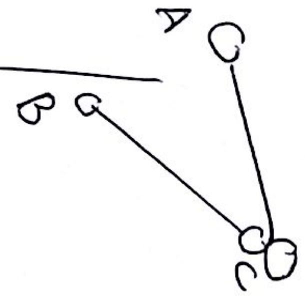
stiffness Matrix

D.o.f

② Load Vector

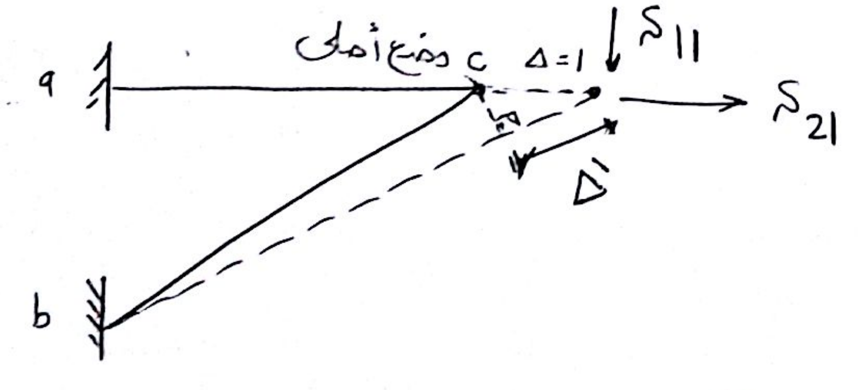
$$P = \begin{bmatrix} 10 \\ -15 \end{bmatrix}$$

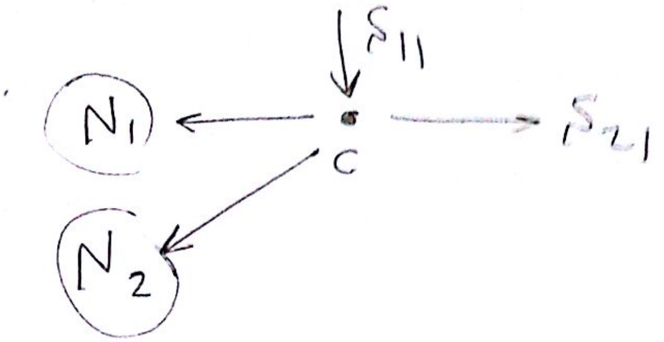




③ Stiffness Matrix [S]

Mode ① $\Delta_x = 1$, $\Delta_y = 0$





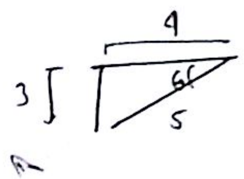
$$P = K \cdot \Delta$$

$$K \rightarrow \frac{EA}{L}$$

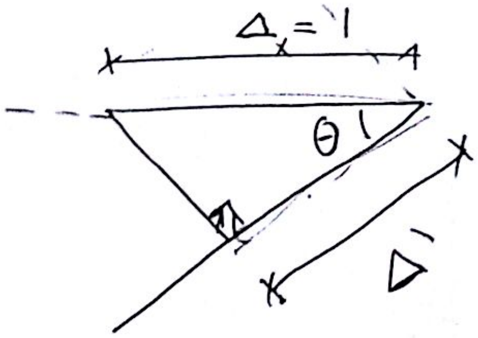
For AC Member

$$* \quad N_1 = \frac{EA}{L} \cdot \Delta_x = \frac{EA}{4} * \underbrace{\Delta_x}_{1}$$

$$\boxed{N_1 = \frac{EA}{4}}$$



For BC Member



$$* \quad N_2 = \frac{EA}{L} \cdot \Delta'$$

$$N_2 = \frac{2EA}{5} * \underbrace{\Delta_x}_{1} \cos\theta$$

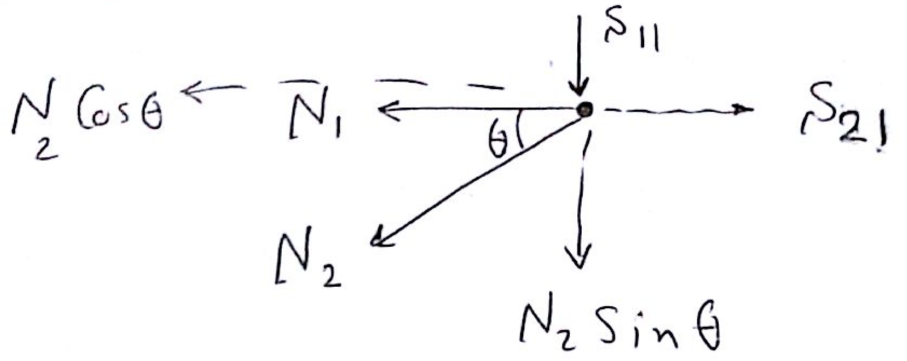
$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

$$N_2 = \frac{2EA}{5} * \frac{4}{5}$$

$$\Delta' = \underbrace{\Delta_x}_{1} \cos\theta$$

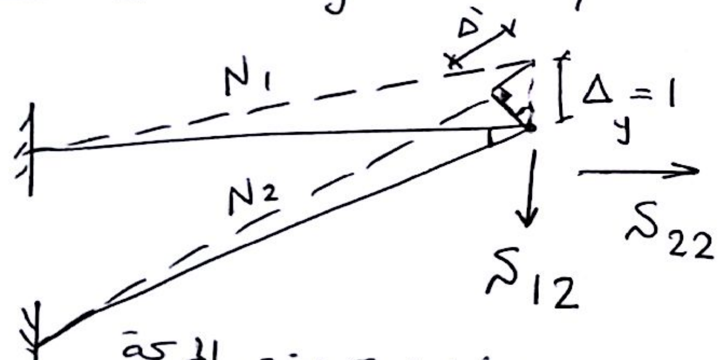
$$\boxed{N_2 = \frac{8EA}{25}}$$

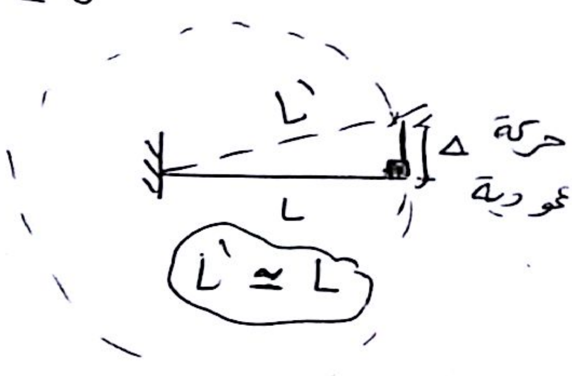


$$\delta_{11} = -N_2 \sin\theta = \frac{8EA}{25} * \frac{3}{5} = 0.506\,EA$$

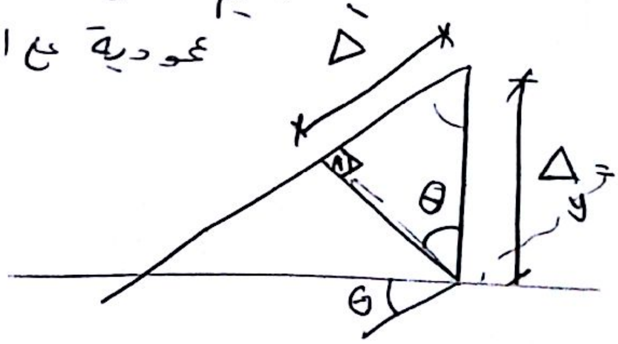
$$\delta_{21} = N_1 + N_2 \cos\theta = \frac{EA}{4} + \frac{8EA}{25} * \frac{4}{5} = 0.192\,EA$$

Mode 2 $\quad \Delta_y = 1 \quad , \quad \Delta_x = 0$





* يتم اهمال أى تغير فى الطول لو كانت الحركة عمودية مع العنصر باعتبار انه نصف قطر فى دائرة ...



$$N_1 = \frac{EA}{L} \cdot \cancel{\Delta}_0 = 0$$

$$N_2 = \frac{EA}{L} \cdot \Delta'$$

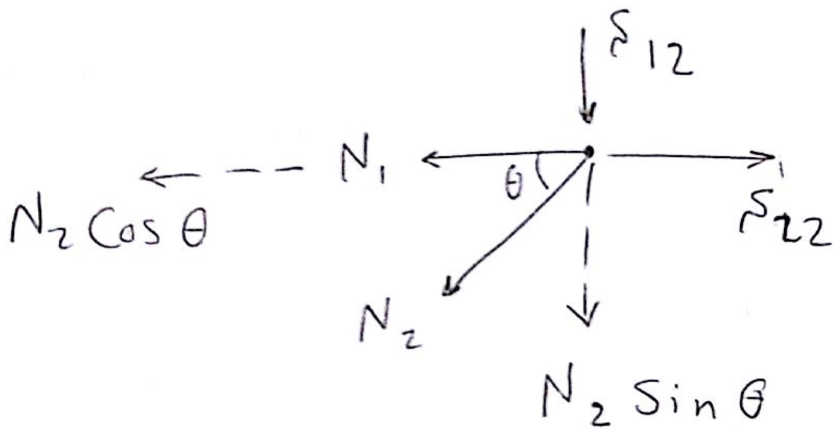
$$N_2 = \frac{2EA}{5} * \frac{3}{5} = \frac{6EA}{25}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

$$\boxed{\Delta' = \Delta_y \sin\theta}$$

Scanned by CamScanner



$$\delta_{22} = \cancel{N_1}^{\,0} + N_2 \cos\theta = \frac{6EA}{25} * \frac{4}{5} = 0.144\,EA$$

$$\delta_{12} = N_2 \sin\theta = \frac{6EA}{25} * \frac{3}{5} = 0.192\,EA$$

$$\boxed{\delta_{12} = \delta_{21}}$$

$$\begin{array}{ccc} & \text{Mode (1)} & \text{Mode (2)} \\ \delta = & \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} & \end{array}$$

القطر

القطر لا عكس -ve

أساسي

صفر

القارماك الأساسية

$$\delta = \begin{bmatrix} 0.506 & \boxed{0.192} \\ \boxed{0.192} & 0.144 \end{bmatrix} EA$$

$$[P] = [S][D]$$

$$\begin{bmatrix} 10 \\ -15 \end{bmatrix} = \begin{bmatrix} 0.506 & 0.192 \\ 0.192 & 0.144 \end{bmatrix} EA * \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\Delta x = \frac{120}{EA}$$

$$\Delta y = \frac{-264.17}{EA}$$

Displacement @ C

Forces

$$N_{final} = N_{mode\ 1} D_1 + N_{mode\ 2} D_2$$

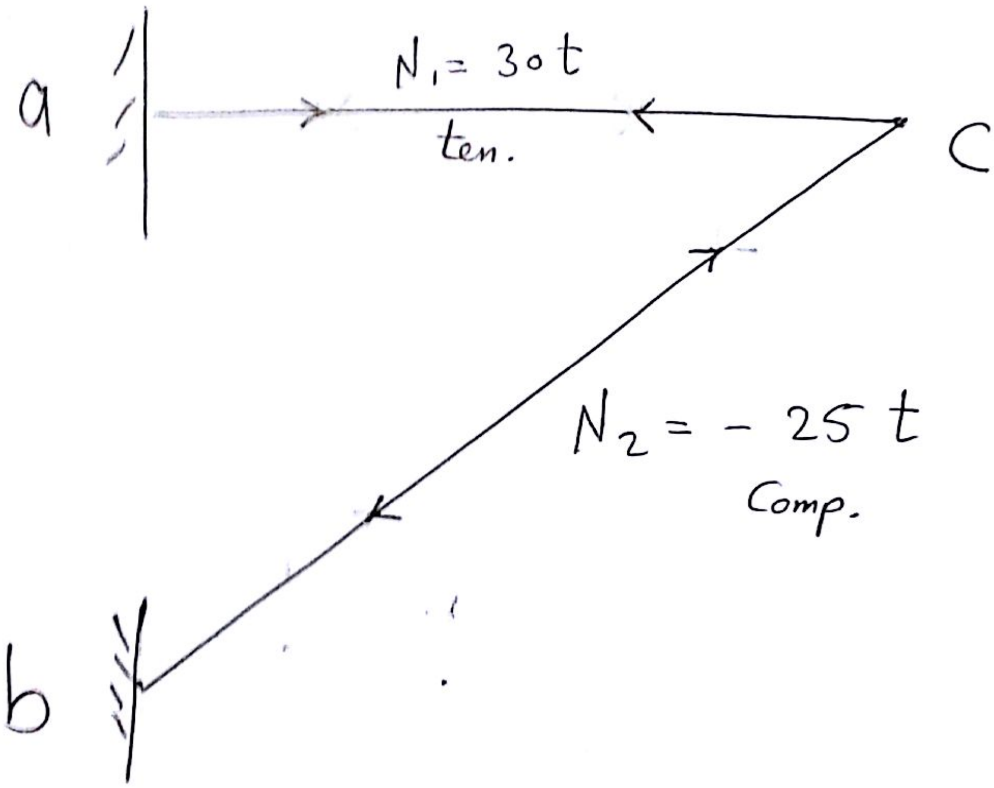
$$N_1 = \frac{\cancel{EA}}{4} * \frac{120}{\cancel{EA}} + \cancel{0 * \frac{-264.17}{EA}}$$

$$\boxed{N_1 = 30 \text{ ton}}$$ tension +ve

$$N_2 = \frac{8\cancel{EA}}{25} * \frac{120}{\cancel{EA}} + \frac{6\cancel{EA}}{25} * \frac{-264.17}{\cancel{EA}}$$

$$\boxed{N_2 = -25 \text{ ton}}$$ Comp. -ve

N.F.D



* في حالة إن (a) ، (b) يسمحوا بالحركة يعني عندهم D.o.f ← هنحركهم برده ويدخلوا معانا .

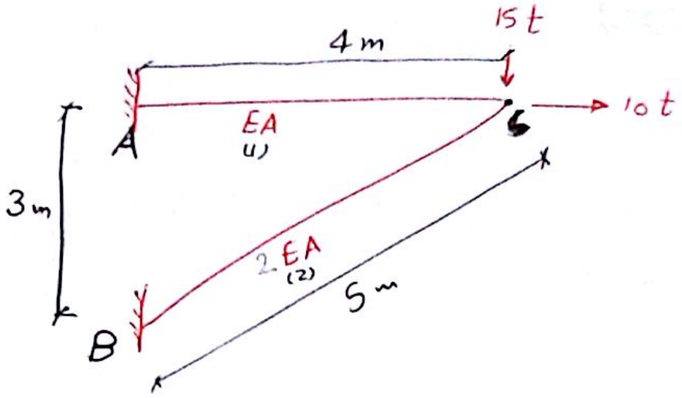
* في حالة التماثل يتم أخذ نصف الشكل ويكون العنصر المشترك بنصف جساءته $\frac{EI}{2}$

18

Direct stiffness — Truss

Calculate Displacement at $\boxed{C}$

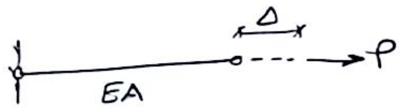
& Forces in members (1&2)



Truss member ⇐ هو عبارة عن سوست spring تقريباً وهو قابل

للانضغاط والانكماش ومقاومته معتمدة عن مساحة مقطعه وليس I

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{P}{A}$$



$$\text{strain} = \frac{\Delta \ \text{الاستطالة}}{L \ \text{الطول الأصلي}}$$

$$\text{Stress} = \underbrace{E}_{\text{معامل المرونة}} \cdot \text{strain} \Rightarrow \frac{P}{A} = E \times \frac{\Delta}{L}$$

$$\therefore P = \left(\frac{EA}{L}\right) \Delta$$

هذه هي المقاومة

spring في الـ k

$$P = \underbrace{K}_{\text{المقاومة}} \Delta$$

أي أن Truss member عبارة عن Spring

أي أن D.o.f ستكون الإزاحة (Δ)

---

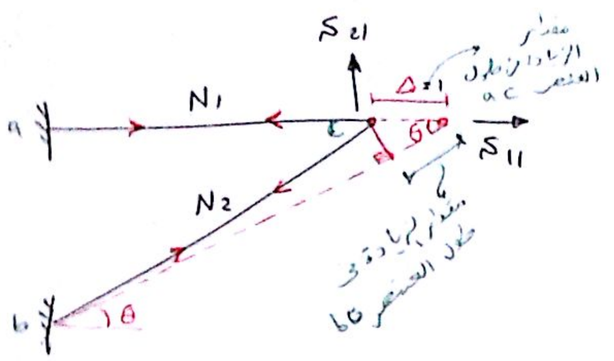
① **D.o.f:** Δ_{c_x} , Δ_{c_y}

$$[D] = \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}$$

② [S]

MODE (1):

$\Delta_x = 1 \quad \& \quad \Delta_y = 0$



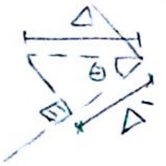
* علشان أجيب قيم المقاومات
لازم أجيب قيم القوى في الجمالون

* وعلشان أجيب القوى في كل عنصر
لازم أعرف Δ بتاعته كام

$$P = K \cdot \Delta$$

$$N_1 = K \cdot \Delta = \left(\frac{EA}{L}\right)_1 * 1 = \boxed{\frac{EA}{4}}$$

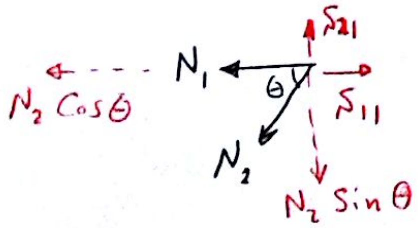
$$N_2 = K \cdot \Delta' = \left(\frac{EA}{L}\right)_2 * \Delta \cos\theta = \frac{2EA}{5} \cos\theta$$



$$= \frac{2EA}{5} * \frac{4}{5} = \boxed{\frac{8EA}{25}}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$



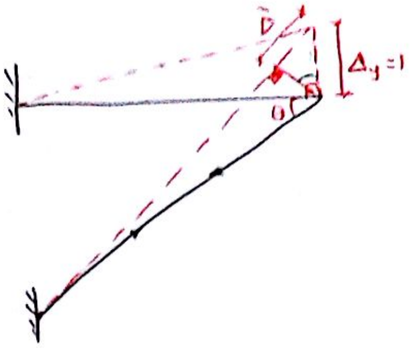
⇐ من اتزان النقطة

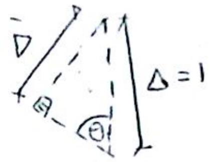
$$S_{11} = N_1 + N_2\cos\theta = \frac{EA}{4} + \frac{2EA}{5}\cos^2\theta = \frac{EA}{4} + \frac{8EA}{25} * \frac{4}{5} = 0.506EA$$

$$S_{21} = N_2 \sin\theta = \frac{8EA}{25} * \frac{3}{5} = \frac{24EA}{125} = 0.192\,EA$$

MODE (2)

$\Delta_x = 0 \quad , \quad \Delta_y = 1$





$\sin\theta = \frac{\Delta'}{\Delta}$

$\Delta' = \Delta \sin\theta$

$N_1 = k \cdot \Delta$ مقدار الاستطالة صغير جداً يؤول إلى الصفر $\Rightarrow N_1 = 0$

[دائماً في العناصر التي تكون تحركها (Δ) عمودي اتجاهها الأصلي]

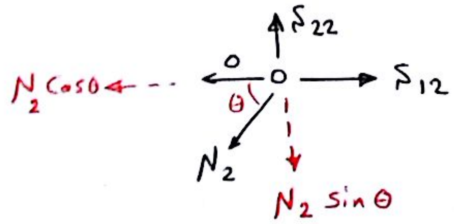
$L' \simeq L$ تقريباً

يعني محصلش استطالة Δ

$N_2 = k \cdot \Delta' = \left(\frac{2EA}{5}\right) \sin\theta$

$= \frac{2EA}{5} * \frac{3}{5} = \frac{6EA}{25}$

⟸ من اتزان النقطة :



N_2

$\therefore S_{12} = \frac{6EA}{25} * \frac{4}{5} = 0.192\, EA \rightarrow \left[\left(\frac{2EA}{5}\right)\sin\theta\right]\cos\theta$

$\therefore S_{22} = \frac{6EA}{25} * \frac{3}{5} = 0.144\, EA \rightarrow \left[\left(\frac{2EA}{5}\right)\sin\theta\right]\sin\theta$

$\left(\frac{2EA}{5}\right)\sin^2\theta$

$$\therefore S = EA \begin{bmatrix} 0.506 & 0.192 \\ 0.192 & 0.144 \end{bmatrix}$$

ملحوظة هامة :

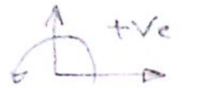
* مما سبق نلاحظ أن المقاومات الأساسية S_{11} & S_{22}

عبارة عن المقاومة مضروبة فى مربع جيب / جيب تمام الزاوية

لذا السرعة سيتم حسابها مباشرة دون تحليل فى حل المسائل...

---

③ **Load Vector:**



$$A_c = P = \begin{bmatrix} 10 \\ -15 \end{bmatrix}$$

هنا مفيش عزوم فمفيش F.E.M الحمل هو اللى على النقطة علطول .

---

④ **Get Displacement:**

$$P = K \cdot \Delta \Rightarrow [A_c] = [S][D]$$

$$\therefore \begin{bmatrix} 10 \\ -15 \end{bmatrix} = \begin{bmatrix} 0.506 & 0.192 \\ 0.192 & 0.144 \end{bmatrix} EA * \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\therefore 10 = (0.506 \Delta_x + 0.192 \Delta y) EA$$

$$\therefore -15 = (0.192 \Delta x + 0.144 \Delta y) EA$$

$$\therefore \Delta x = \frac{120}{EA} \quad \& \quad \Delta y = \frac{-264.17}{EA}$$

(5) Get Forces

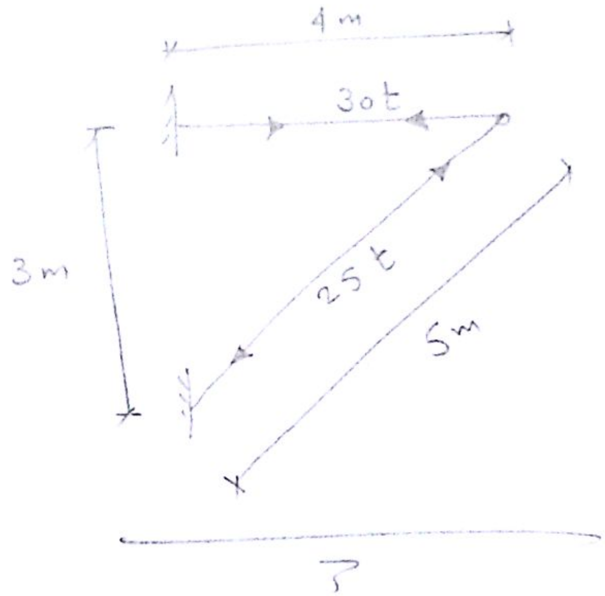
$$N_{Final} = N_{mode①} D_1 + N_{mode②} D_2$$

$$N_{1\,Final} = F_1 = \frac{\cancel{EA}}{4} * \frac{\overset{\Delta_x}{120}}{\cancel{EA}} + 0 * \frac{-264.17}{EA} = 30 \text{ ton}$$

(Tension)

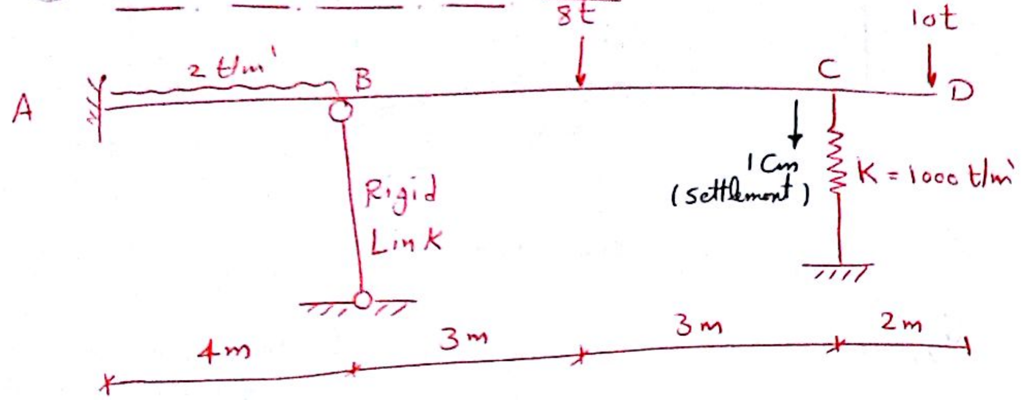
$$N_{2\,Final} = F_2 = \frac{8\cancel{EA}}{25} * \frac{120}{\cancel{EA}} + \frac{6\cancel{EA}}{25} * \frac{-264.17}{\cancel{EA}} \simeq -25 \text{ ton}$$

(Compression)

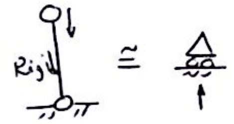


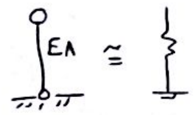
Scanned by CamScanner

Ex ② Direct stiffness (Beams)



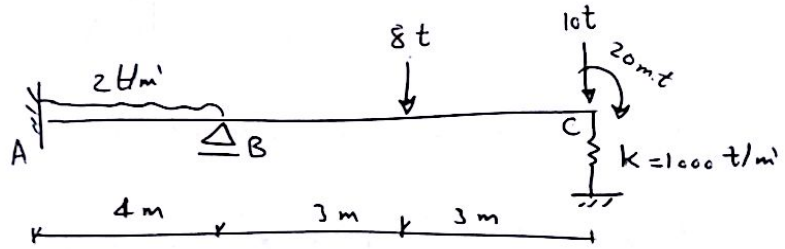
Note that:

Rigid Link ≅ Roller ⇒  ⇒ $\Delta_y = 0$

flexable Link (EA) ≅ spring ⇒  ⇒ $\Delta_y = ✓$

[Truss member] (Axial)

Modified Beam:

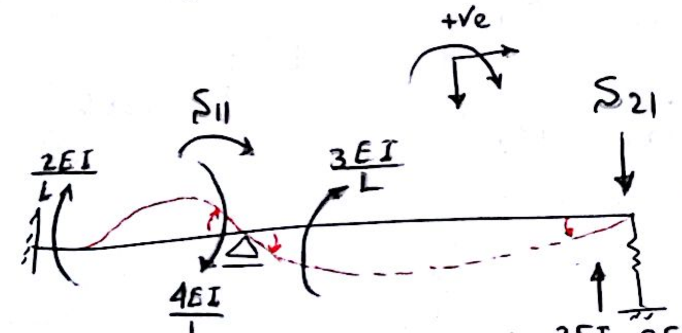


① D.o.f:

$$\theta_B \ , \ \Delta_{c_y} \quad \Rightarrow [D] = \begin{bmatrix} \theta_B \\ \Delta_c \end{bmatrix}$$

② [S]

Mode ①



ملهوش لازمة

$\frac{\Delta M}{L} = \frac{6EI}{L}$ $\quad \frac{6EI}{L}$

$\frac{\Delta M}{L} = \frac{3EI}{L^2} = \frac{3EI}{6^2}$

علشان أقدر أجيب Δ ٢،١ لازم المقاومة بتاعة مقاومة قوى يكون عندى Shear وليست عزم لذا نحسب ال

Δ عند نقطة ٥٥ بس

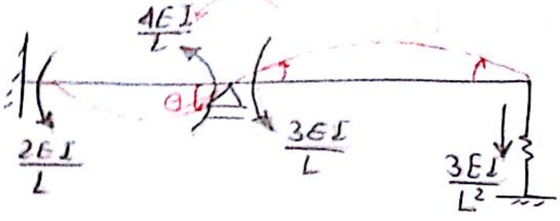
$$S_{11} = \frac{4EI}{4} + \frac{3EI}{6} = 1.5 \, EI$$

Shear
قص
Moment

$$S_{21} = -\frac{3EI}{6^2} = 0.083 \, EI$$

(لأنها حركة مقاومة مش دوران)

Mode 1



نفس ال Mode لكن بفرض عكس الاتجاهات لو جبت

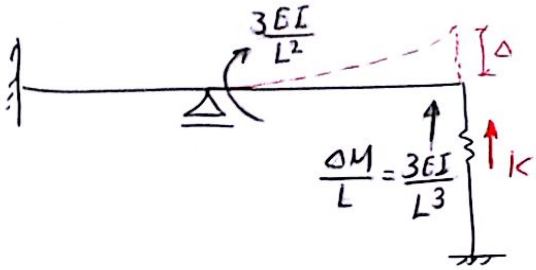
$$S_{11} = \frac{4EI}{4} + \frac{3EI}{6} = 1.5\ EI$$

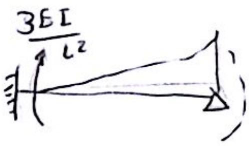
$$S_{21} = -\frac{3EI}{L^2} = -0.083\ EI$$

(Shear مش ← Moment)
علشان ده مقاومة
حركة رأسية

عكس قاعدة الاشارات

Mode 2





$$S_{12} = -\frac{3EI}{6^2} = -0.083\ EI$$

$$S_{22} = +\frac{3EI}{6^3} + K * \frac{EI}{EI} = \frac{EI}{72} + \frac{1000\ EI}{6000} = 0.181\ EI$$

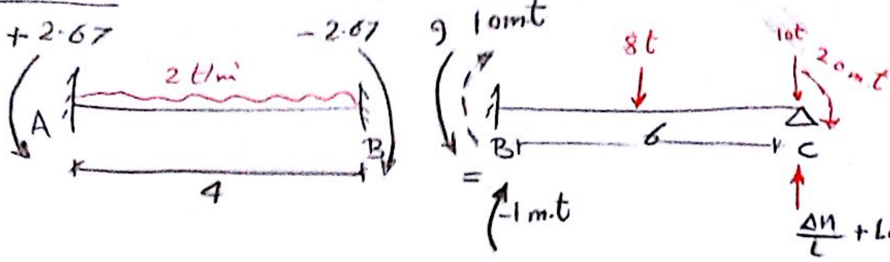
علشان نعرف نجمع

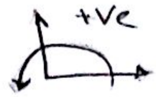
EI

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = EI \begin{bmatrix} 1.5 & -0.083 \\ -0.083 & 0.181 \end{bmatrix}$$

③ F.E.M

$\frac{3PL}{16} = 9\ m.t$



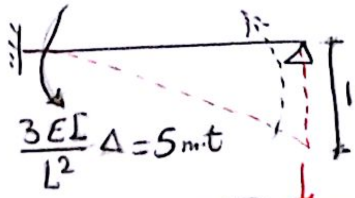


$\frac{\Delta M}{L} + \text{Load} = \frac{20+1}{6} + \frac{8}{2} = 7.5\ t$

1 Cm

$\frac{3EI}{L^2}\Delta = \frac{3 \times 6000}{6^2} \times 0.01$

$= 5\ m.t$



نتيجة الهبوط

$\frac{3EI}{L^2}\Delta = 5\ m.t$

$\frac{\Delta M}{L} = \frac{3EI}{L^3}\Delta = 0.833$

$$A_{F.EM} = \begin{bmatrix} 1.33 \\ 6.667 \end{bmatrix}$$

$-2.67 - 1 + 5$

$7.5 - 0.833$

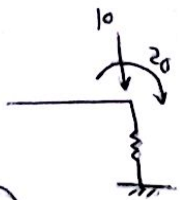
④ Load Vector $[A_c]$:

$$A_c = A_J - A_{F.E.M}$$

$$A_J = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

من شكل الكمرة بعدلة

ناخذ الي بيحمل Δ



$$\therefore [A_c] = \begin{bmatrix} 0 \\ -10 \end{bmatrix} - \begin{bmatrix} 1.33 \\ 6.667 \end{bmatrix} = \begin{bmatrix} -1.33 \\ -16.667 \end{bmatrix}$$

⑤ Get Displacement [D]

$$P = \Delta . K$$

$$[A_c] = [D].[S]$$

$$\begin{bmatrix} -1.33 \\ -16.667 \end{bmatrix} = EI \begin{bmatrix} 1.5 & -0.083 \\ -0.083 & 0.181 \end{bmatrix} * \begin{bmatrix} \theta_b \\ \Delta_c \end{bmatrix}$$

نحلها بالطول ع الآلـة أو نضرب صف ف عمود [D]

$$-\frac{1.33}{EI} = [1.5\,\theta_b - 0.083\,\Delta_c]\,EI$$

$$-\frac{16.667}{EI} = [-0.083\,\theta_b + 0.181\,\Delta_c]\,EI$$

لحل المعادلتين :

$$\therefore \theta_b = \frac{-6.13}{EI} \quad \text{و} \quad \Delta_c = \frac{-94.9}{EI}$$

---

(5) Moments:

$$M_{Final} = M_{F.E.M} + \underbrace{M_{mode(1)}}_{\text{تابع}}\,\theta_b + M_{mode(2)}\,\Delta_c$$

$M_{ab} \rightarrow$ mode θ_b $\quad$ $M_{ab} \rightarrow$ mode Δ_c

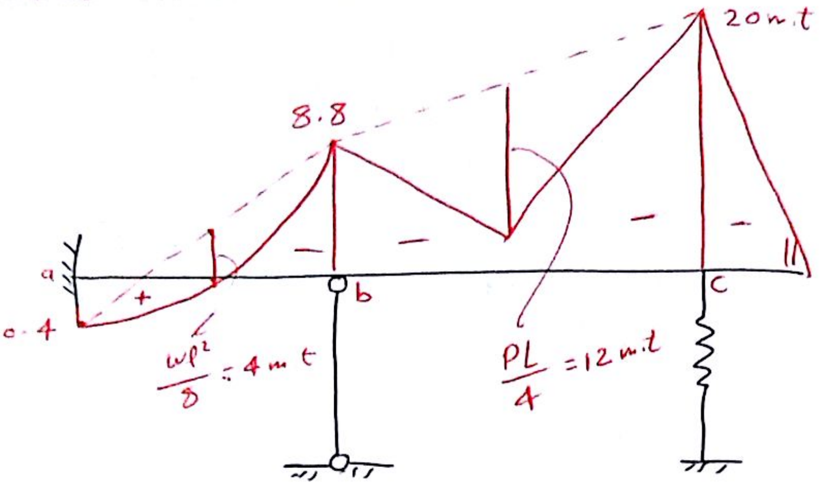
$$* \; M_{ab} = 2.67 + \frac{2EI}{4} * \frac{-6.13}{EI} + 0 * \frac{-94.9}{EI} = -0.4 \text{ m.t}$$

عند ab

$$* \; M_{ba} = -2.67 + \frac{4EI}{4} * \frac{-6.13}{EI} + 0 = -8.8 \text{ m.t}$$

(5-1)

$$* \; M_{bc} = 4 + \frac{3EI}{6} * \frac{-6.13}{EI} + \left(-\frac{3EI}{6^2}\right) * \frac{-94.9}{EI} = +8.8 \text{ m.t}$$



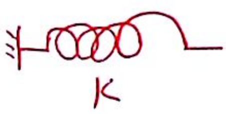
B.M.D

* لو طلب S.F.D برسم Free Body Diagram وجيب منه (R)

* مقاومة (K) Spring لا تظهر إلا مرة واحدة في حالة

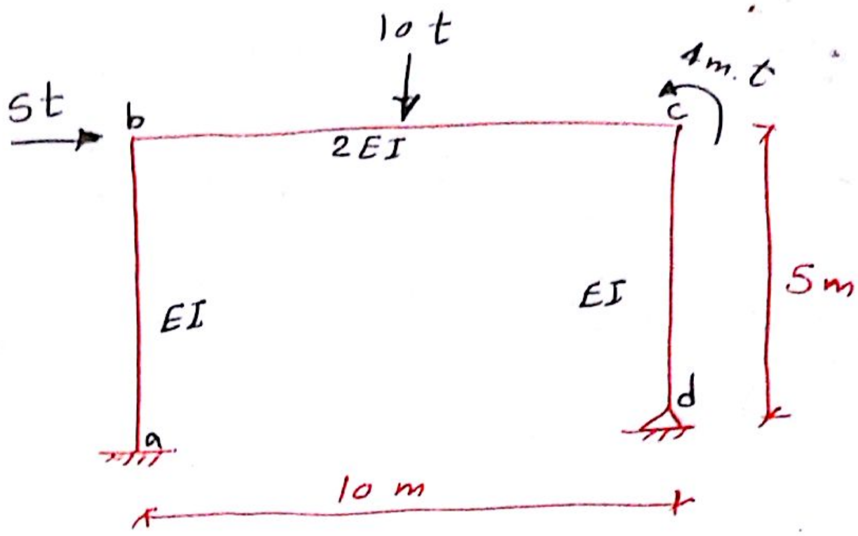
Δ عندها .  $D.o.f = \Delta$

* مقاومة (K) Radial Spring تظهر في حالة وجود θ عنده

θ  $D.o.f = \theta$

*  عندها $D.o.f = \Delta$ بس

* Direct stiffness [Frames]



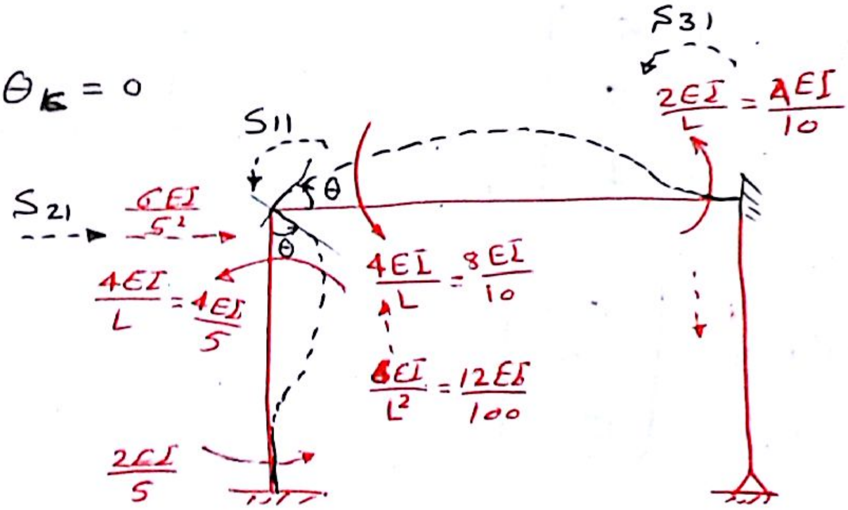
① D.o.f:

$$D = \begin{bmatrix} \theta_b \\ \Delta_b \\ \theta_c \end{bmatrix}$$

② [S]

MODE①:

$\theta_b = 1 \quad , \quad \Delta_b = \theta_c = 0$



* $S_{11} = \frac{\overset{4}{\cancel{8}}EI}{\underset{5}{\cancel{10}}} + \frac{4EI}{5}$

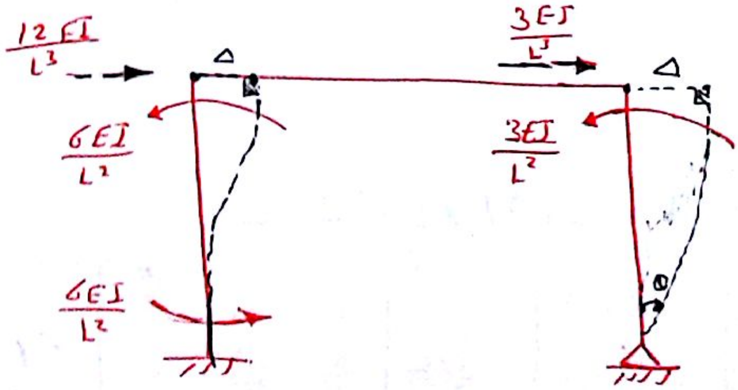
$= \frac{8EI}{5}$

* $S_{21} = \frac{6EI}{25}$

* $S_{31} = \frac{2EI}{5}$

MODE②:

$\Delta_b = 1 \quad , \quad \theta_b = \theta_c = 0$



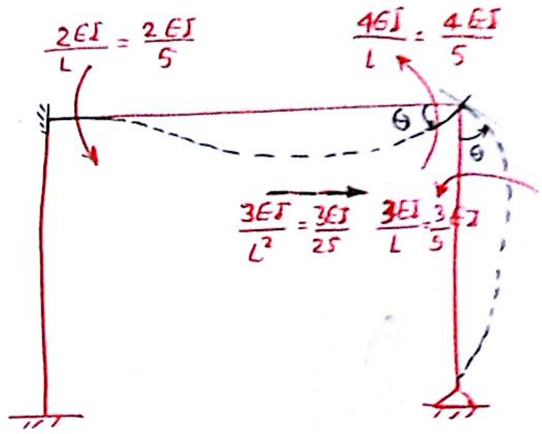
* $S_{12} = \frac{6EI}{L^2} = \frac{6EI}{25}$

* $S_{22} = \frac{12EI}{L^3} + \frac{3EI}{L^3} = \frac{12EI}{5^3} + \frac{3EI}{5^3} = \frac{15EI}{125} = \frac{3}{25}EI$

* $S_{32} = \frac{3EI}{L^2} = \frac{3EI}{25}$

MODE (3):

$\theta_c = 1 \;,\; \theta_b = \Delta_b = 0$



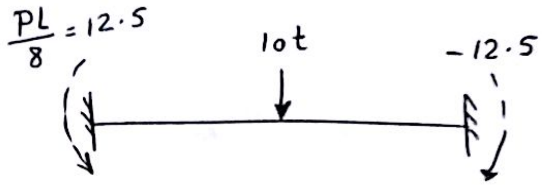
$S_{13} = \frac{2EI}{5}$

$S_{23} = \frac{3EI}{25}$

$S_{33} = \frac{7EI}{5}$

(3) $A_c = A_J - A_{F.E.M}$

$$\vec{A_J} = \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$$



$$A_{F.E.M} = \begin{bmatrix} 12.5 \\ 0 \\ -12.5 \end{bmatrix}$$

$$\therefore A_c = \begin{bmatrix} -12.5 \\ 5 \\ 16.5 \end{bmatrix}$$

---

(4) [D]

$A_c = [S] * [D]$

$$\begin{bmatrix} -12.5 \\ 5 \\ 16.5 \end{bmatrix} = \begin{bmatrix} \frac{8}{5} & \frac{6}{25} & \frac{2}{5} \\ \frac{6}{25} & \frac{15}{125} & \frac{3}{25} \\ \frac{2}{5} & \frac{3}{25} & \frac{7}{5} \end{bmatrix} EI \begin{bmatrix} \theta_b \\ \Delta_b \\ \theta_c \end{bmatrix} \Rightarrow \begin{array}{l} \theta_b = \frac{-21.76}{EI} \\ \Delta_b = \frac{+73.5}{EI} \\ \theta_c = \frac{+11.7}{EI} \end{array}$$

(5) M

$$M_{Final} = M_{F.E.M} + M_{mode①}\,\theta_b + M_{mode②}\,\Delta_b + M_{mode③}\,\theta_c$$

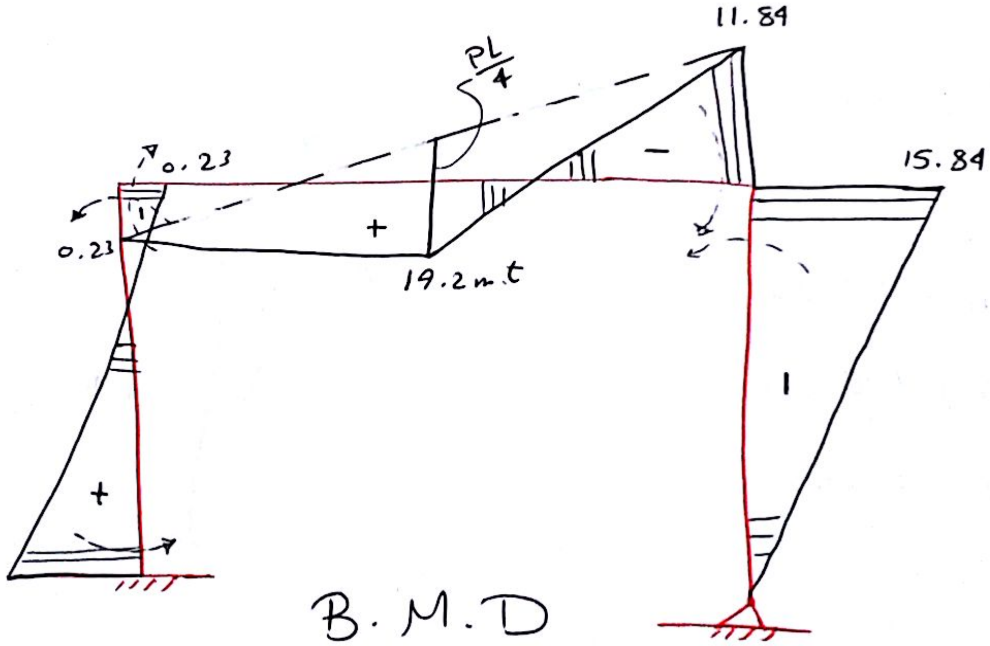
$$M_{ab} = 0 + \frac{2EI}{5} * \frac{-21.76}{EI} + \frac{6EI}{25} * \frac{73.5}{EI} + 0 = +8.936 \text{ m.t}$$

$$M_{ba} = 0 + \frac{4EI}{5} * \frac{-21.76}{EI} + \frac{6EI}{25} * \frac{73.5}{EI} + 0 = +0.23 \text{ m.t}$$

$$M_{bc} = 12.5 + \frac{4EI}{5} * \frac{-21.76}{EI} + 0 + \frac{2EI}{5} * \frac{11.7}{EI} = -0.23 \text{ m.t}$$

$$M_{cb} = -11.84 \text{ m.t}$$

$$M_{cd} = +15.84 \text{ m.t}$$

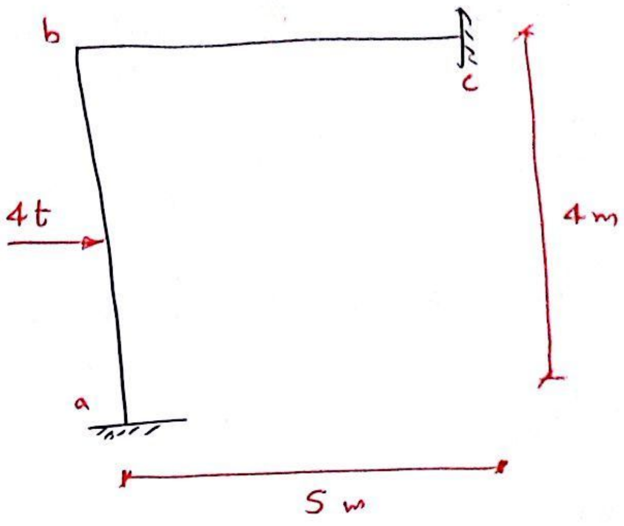


B.M.D

Consider Axial Deformation

$EI = 10000$

$EA = 5000$



① D.o.F

فى حالة اعتبار Axial Deformation يعتبر العنصر

مثل Truss حيث أن مقاومته للتشوه

تعتمد على قطاعه $\frac{EA}{L}$

$$D = \begin{bmatrix} \theta_b \\ \Delta_H \\ \Delta_V \end{bmatrix}$$

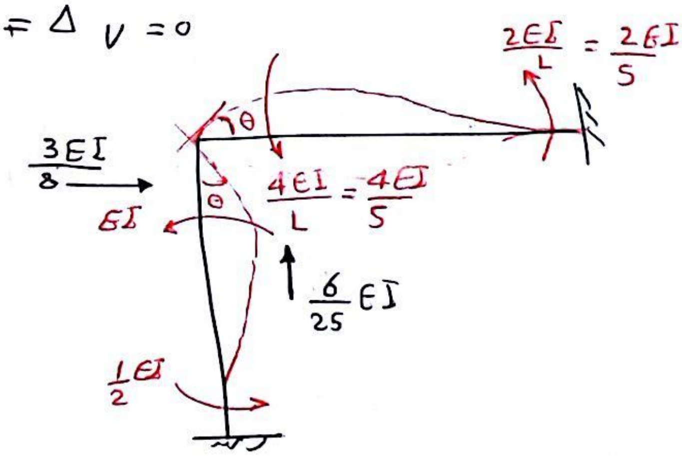
② [S]

MODE ① $\theta_b = 1, \Delta_H = \Delta_V = 0$

$S_{11} = 1.8\,EI$

$S_{21} = \frac{3}{8}EI$

$S_{31} = 0.24\,EI$

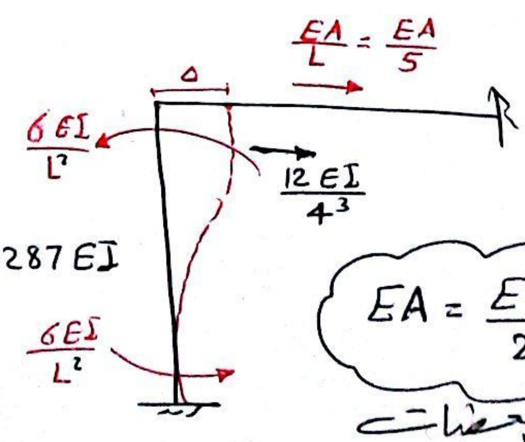


MODE ② $\Delta_H = 1, \theta_b = \Delta_V = 0$

$S_{12} = \frac{6EI}{4^2} = \frac{3EI}{8}$

$S_{22} = \frac{3EI}{16} + \frac{EA}{5} = \frac{3EI}{16} + \frac{EI}{10} = 0.287\,EI$

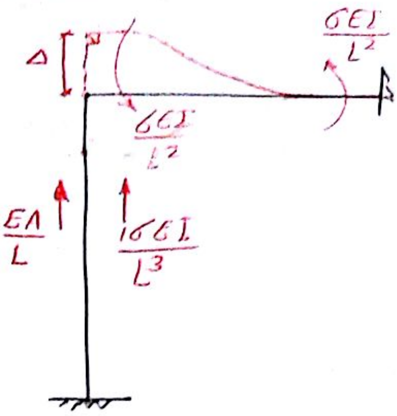
$S_{32} = 0$



$EA = \frac{EI}{2}$

من المعطيات

MODE ③



$S_{13} = \frac{6EI}{5^2} = 0.24\,EI$

$S_{23} = 0$

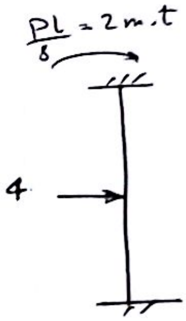
$S_{33} = \frac{16EI}{4^3} + \frac{EA}{L} = \frac{EI}{4} + \frac{EI}{8}$

$= \frac{3}{8}EI$

$$S = \begin{bmatrix} 1.8 & \frac{3}{8} & 0.24 \\ \frac{3}{8} & 0.287 & 0 \\ 0.24 & 0 & \frac{3}{8} \end{bmatrix} EI$$

③ A_c

$A_J = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad A_{F.E.M} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$



$A_c = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

④ [D]

$$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} EI \times \begin{bmatrix} \theta_b \\ \Delta_H \\ \Delta_V \end{bmatrix}$$

$\theta_b = \frac{\quad}{EI}$

$\Delta_H = \frac{\quad}{EI}$

$\Delta_V = \frac{\quad}{EI}$